Section 7.5. Euler and Hamilton Paths

Note. In this section we consider two classical graph theory problems.

Definition 7.5.1. An *Euler circuit* in a graph G is a simple circuit (i.e., no repeated edges) containing every edge of G. An *Euler path* G is a simple path containing every edge in G.

Theorem 7.5.1. A connected multigraph has an Euler circuit if and only if its vertices has even degree.

Theorem 7.5.2. A connected multigraph has an Euler path (but not an Euler circuit) if and only if it has exactly two vertices of odd degree.

Example. Pages 475 and 476, the Königsberg Bridge Problem.

Definition 7.5.2. A path $x_0, x_1, \dots, x_n$ in $G = (V, E)$ is a *Hamilton path* if $V = \{x_0, x_1, \dots, x_n\}$ and $x_i \neq x_j$ for $i \neq j$. A circuit $x_0, x_1, \dots, x_n, x_0$ is a *Hamiltonian circuit* if $x_0, x_1, \dots, x_n$ is a Hamilton path.

Theorem 7.5.3. If G is a connected simple graph with $n \geq 3$ vertices. Then G has a Hamiltonian circuit if the degree of each vertex is at least $n/2$.

Note. The complete graph K_n has a Hamilton circuit for all $n \geq 3$.

Example. Page 487 Number 40.

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