DISCRETE STRUCTURES
TEST 1 - Spring 2001

NAME _____ STUDENT NUMBER _____

SHOW ALL WORK!!! Be clear and convince me that you understand what is going on. Use all symbols properly. Partial credit will only be given for answers which are properly correct. Each problem is worth 12 points. No calculators!

1. Let p and q be the propositions

p: You drive over 65 miles per hour.

q: You get a speeding ticket.

Write the following proposition using p and q and logical connectives: "You get a speeding ticket, but you do not drive over 65 miles per hour."

2. Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

3. Let $Q(x, y)$ be the statement "$x + y = x - y$." If the universe of discourse for both varaibles is the set of integers, what is the truth value of $\forall x \, \exists y \, Q(x, y)$ and why?

4. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find $A \times B$.

5. Let A, B, and C be sets. Prove that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

6. Let f be the function from $\mathbf{R}$ to $\mathbf{R}$ defined by $f(x) = x^2$. Find $f^{-1}(\{x \mid x > 4\})$.

7. If $S = \{1, 3, 5, 7\}$, then what is the value of $\sum_{j \in S} j^2$?

8. Find the least integer n such that $f(x)$ is $O(x^n)$ for $f(x) = 3x^5 + (\log x)^4$.

Bonus 1. Draw a graph of $f(x) = \lceil 3x - 2 \rceil$.

Bonus 2. Show that if A and B are sets, then $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$.