DISCRETE STRUCTURES

TEST 2 - Spring 2001

NAME ______________________________ STUDENT NUMBER ________________________

SHOW ALL WORK!!! Be clear and convince me that you understand what is going on. Use all symbols properly. Partial credit will only be given for answers which are properly correct. Each problem is worth 12 points. No calculators!

1. Describe an algorithm for finding the smallest integer in a finite sequence of natural numbers. Use *psuedocode* to describe your algorithm (as in the text).

2. The conventional algorithm for evaluating a polynomial $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ at $x = c$ can be expressed in psuedocode by

procedure *polynomial*($c, a_0, a_1, \ldots, a_n$: real numbers)
power$:= 1$
$y := a_0$
for $i := 1$ **to** n
begin
    power$:=$ *power* $* \, c$
    $y := y + a_i \, *$ *power*
end $\{y = a_n c^n + a_{n-1} c^{n-1} + \cdots + a_1 c + a_0\}$

where the final value y is the value of the polynomial at $x = c$. Exactly how many multiplications and additions are used to evaluate a polynomial of degree n at $x = c$? (Do not count additions used to increment the loop variable.)

3. Show that if $n \mid m$, where n and m are positive integers greater than 1, then if $a \equiv b \pmod{m}$, where a and b are integers, then $a \equiv b \pmod{n}$.

4. Suppose $\mathbf{A} = \begin{bmatrix} 0 & -1 \\ 7 & 2 \\ -4 & -3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 4 & -1 & 2 & 3 & 0 \\ -2 & 0 & 3 & 4 & 1 \end{bmatrix}$. Find $\mathbf{AB}$.

5. Let $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find $\mathbf{A} \odot \mathbf{B}$ (that is, the Boolean product of $\mathbf{A}$ and $\mathbf{B}$).

6. Give an indirect proof (that is, a proof of the contrapositive) of "If n is an integer and $n^3 + 5$ is odd, then n is even.

7. Use mathematical induction to prove that for $r \neq 1$:

$$\sum_{j=0}^{n} ar^j = \frac{ar^{n+1} - a}{r - 1}.$$

8. Give a recursive definition of the sequence $\{a_n\}$ where $a_n = 10^n$.

Bonus 1. Use mathematical induction to prove that if $\mathbf{A} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ then $\mathbf{A} = \begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix}$.

Bonus 2. Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.