



4. Show that if  $n$  and  $k$  are positive integers, then

$$C(n + 1, k) = (n + 1)C(n, k - 1)/k.$$

5. What is the probability that a five-card poker hand contains at least one ace?

6. In a superlottery, players win a fortune if they choose the eight numbers selected by a computer from the positive integers not exceeding 100. What is the probability that a player wins this superlottery?

7. Suppose that the number of bacteria in a colony triples every hour. Set up a recurrence relation for the number of bacteria after  $n$  hours have elapsed. If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?
8. In a survey of 270 college students, it is found that 64 like brussels sprouts, 94 like broccoli, 58 like cauliflower, 26 like both brussels sprouts and broccoli, 28 like both brussels sprouts and cauliflower, 22 like both broccoli and cauliflower, and 14 like all three vegetables. How many of the 270 students do not like any of these vegetables?

**Bonus 1.** Find a recurrence relation for the number of bit strings of length  $n$  that do not contain three consecutive 0s.

**Bonus 2.** What is the coefficient of  $x^{101}y^{99}$  in the expansion of  $(2x - 3y)^{200}$ ?