DISCRETE STRUCTURES

TEST 4 — Spring 2001

NAME ________________________________ STUDENT NUMBER _________________________

SHOW ALL WORK!!! Be clear and convince me that you understand what is going on. Use all symbols properly. Partial credit will only be given for answers which are partially correct! Each problem is worth 12 points.

1. Let R be the relation $\{(1,2),(1,3),(2,3),(2,4),(3,1)\}$ and let S be the relation $\{(2,1),(3,1),(3,2),(4,2)\}$. Find $S \circ R$.

2. Let R_1 and R_2 be relations on a set A represented by the matrices:

$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find the matrix which represents $R_1 \bigcap R_2$.

3. Give a description of each of the congruence classes of the integers modulo 6.

4. State the definition of *simple graph*.

5. Determine whether the graph is bipartite and explain:

6. Find the adjacency matrix of the given directed multigraph:

7. Find the number of paths of length $n = 4$ between two different vertices in K_4.

8. Determine whether the directed graph has an Euler circuit and explain:

Bonus 1. Find the smallest equivalence relation on the set $\{a, b, c, d, e\}$ containing the relation $\{(a, b), (a, c), (d, e)\}$.

Bonus 2. Show that if a simple graph G has k connected components and these components have $n_1, n_2, \ldots, n_k$ vertices, respectively, then the number of edges of G does not exceed $\sum_{i=1}^{k} C(n_i, 2)$.