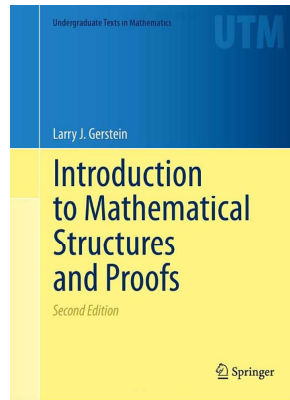


Mathematical Reasoning

Chapter 2. Sets

2.6. Indexed Sets.—Proofs of Theorems



Theorem 2.32(a)

Theorem 2.32. Let A be a set, $\{B_i\}_{i \in I}$ be an indexed family of sets, and let U be the universal set. Then:

$$(a) \quad A - \bigcap_{i \in I} B_i = \bigcup_{i \in I} (A - B_i)$$

Proof. We give a sequence of double implications:

$$\begin{aligned} x \in A - \bigcap_{i \in I} B_i &\Leftrightarrow x \in A \text{ and } x \notin \bigcap_{i \in I} B_i \\ &\Leftrightarrow x \in A \text{ and } (\exists i \in I)(x \notin B_i) \text{ by Note 2.6.A} \\ &\Leftrightarrow (\exists i \in I)(x \in A \text{ and } x \notin B_i) \\ &\Leftrightarrow (\exists i \in I)(x \in A - B_i) \\ &\Leftrightarrow x \in \bigcup_{i \in I} (A - B_i). \end{aligned}$$

So the elements of $A - \bigcap_{i \in I} B_i$ are the same as the elements of $\bigcup_{i \in I} (A - B_i)$ and hence the sets are equal, as claimed. \square