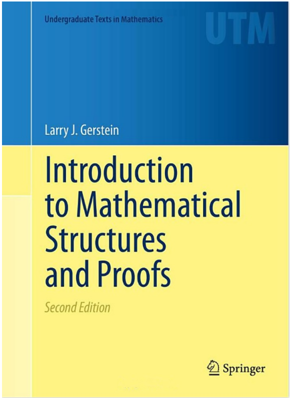
Mathematical Reasoning

Chapter 2. Sets

2.6. Indexed Sets.—Proofs of Theorems



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Theorem 2.32(a)

Theorem 2.32. Let A be a set, $\{B_i\}_{i \in I}$ be an indexed family of sets, and let U be the universal set. Then:

(a) $A - \cap_{i \in I} B_i = \cup_{i \in I}(A - B_i)$

Proof. We give a sequence of double implications:

$$
\begin{aligned}
x \in A - \cap_{i \in I} B_i \quad &\Leftrightarrow \quad x \in A \text{ and } x \notin \cap_{i \in I} B_i \\
&\Leftrightarrow \quad x \in A \text{ and } (\exists i \in I)(x \notin B_i) \text{ by Note 2.6.A} \\
&\Leftrightarrow \quad (\exists i \in I)(x \in A \text{ and } x \notin B_i) \\
&\Leftrightarrow \quad (\exists i \in I)(x \in A - B_i) \\
&\Leftrightarrow \quad x \in \cup_{i \in I}(A - B_i).
\end{aligned}
$$

So the elements of $A - \cap_{i \in I}$ are the same as the elements of $\cup_{i \in I}(A - B_i)$ and hence the sets are equal, as claimed. □

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So the elements of $A - \cap_{i \in I}$ are the same as the elements of $\cup_{i \in I}(A - B_i)$ and hence the sets are equal, as claimed. □