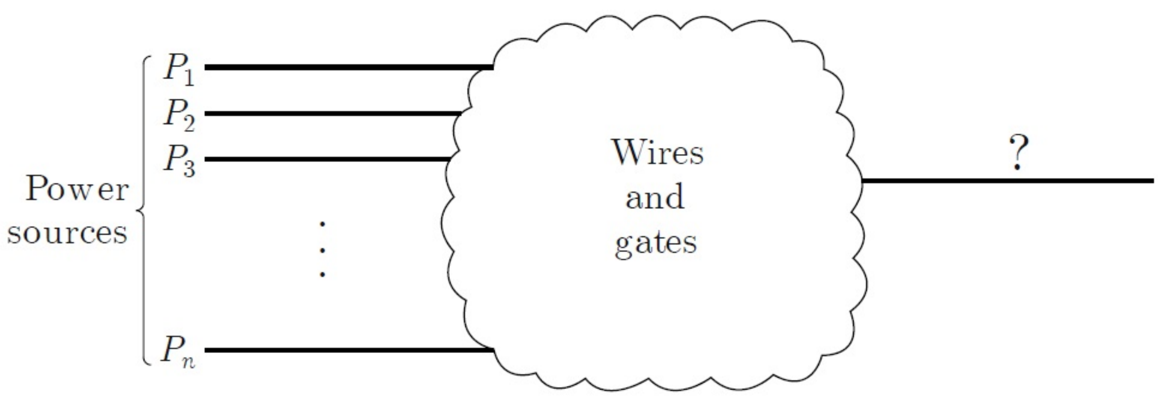
1.6. Application: A Brief Introduction to Switching Circuits

Note. In this section we use sentential forms to represent electronic switching circuits. At a fundamental level, a computer is a complicated collection of switching circuits.

Note. We consider a collection of wires connected to a collection of power sources. By convention, the wires are represented by lines coming in from the left side of a diagram. Power may or may not be flowing through a given wire connected to a power source (that is, the power supply may be "on" or "off"). These wires (and others further to the right) are linked together by switching devices called *gates*. The gates have one or two wires entering from the left side and one wire exiting to the right. Ultimately only one wire exits on the right. Whether power is off or on in this one wire is determined by the inputs from the left (the on/off status of the power sources on the left) and the configuration of the gates.



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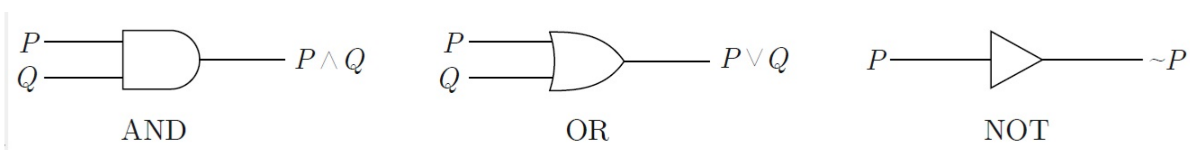
Note. The two basic problems of this section are (1) to determine, based on the on/off status of the power sources on the left, whether or not power flows out of the single wire on the right side, and (2) to find a configuration of a minimal number of gates that will result in the same output to the right (based on the input from the left) as a given network of gates. We deal with these problems by relating them to sentential forms and sentential variables. The wires on left become the variables and on/off is associated with T/F. A truth table then allows us to solve problem (1) and if we can find a logically equivalent form of the sentential form with the fewest logical connectives then we solve problem (2).

Definition. We consider the basic gates AND, OR, and NOT which we symbolically denote as follows (along with the associated sentential forms):



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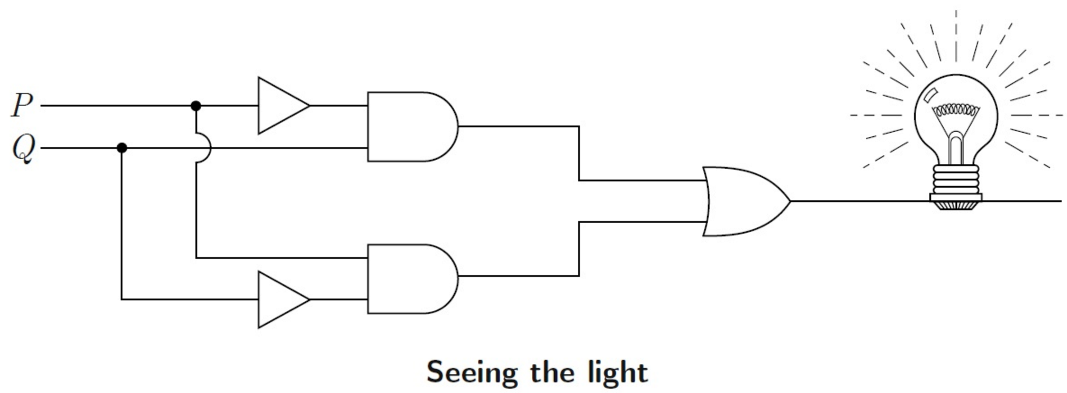
Example 1.36. A room light is to be connected to switches in two locations, and a flick of either switch should change the state of the light (turns it off when on, and vice versa). Design a circuit for this purpose, using AND, OR, and NOT gates.

Solution. Let P and Q denote the wires from the two wall switches (in terms of actual wiring, we are trying to complete a circuit, so you might think of the wire on the right side as connected to the other terminal of the common power source to which the switches are attached). Let S be a sentential form that satisfies the desired description of the switches. Say we have both P and Q is the state F (off) so that the light is off and S is F. By changing the truth value of either P once or changing Q once (but not both) should then result in changing S to T. If we change the truth value of both P and Q (which corresponds to flipping both switches) then the truth value of S should remain F. This implies the following truth table:

P	Q	S
F	F	F
F	T	T
T	F	T
T	T	F

So S is T when P and Q have different truth values. This occurs when $\sim P \wedge Q$ is T (from line 2 of the truth table) or when $P \wedge \sim Q$ is T (from line 3); otherwise S is F. So we need either $\sim \wedge Q$ or $P \wedge \sim Q$ to be T. So we have $S \equiv (\sim P \wedge Q) \vee (P \wedge \sim Q)$. To create a diagram of gates, we start with inputs P and Q on the left. To get $\sim P \wedge Q$, we first feed P into a NOT gate and then feed the output from this gate (namely, $\sim P$) and Q into an AND gate, producing the output of $\sim P \wedge Q$ (in top center of the diagram below). To get $P \wedge \sim Q$, we similarly feed Q into a NOT gate and then feed the output from this gate (namely, $\sim Q$) and P into an AND gate, producing the output of $P \wedge \sim Q$ (in bottom center of the diagram below). Notice that we need to split the lines from both P and Q to perform this type of "wiring." Finally, we feed the outputs $\sim P \wedge Q$ and $P \wedge \sim Q$ into an OR gate; this output is then connected to the light bulb.



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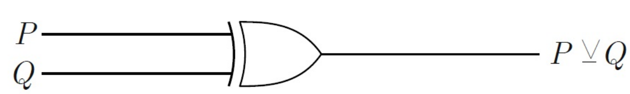
Note/Definition. By Note 1.6.A, we see that the logical connective `xor` can be used to produce a sentential form logically equivalent to sentential form S in Example 1.36: $S = (\sim P \wedge Q) \vee (P \wedge \sim Q) \equiv P \texttt{ xor } Q$. We now denote exclusive disjunction `xor` symbolically as $\underline{\vee}$, so that we can say $S = (\sim P \wedge Q) \vee (P \wedge \sim Q) \equiv P \underline{\vee} Q$. We also introduce this as a gate, the "EXCLUSIVE OR" gate:



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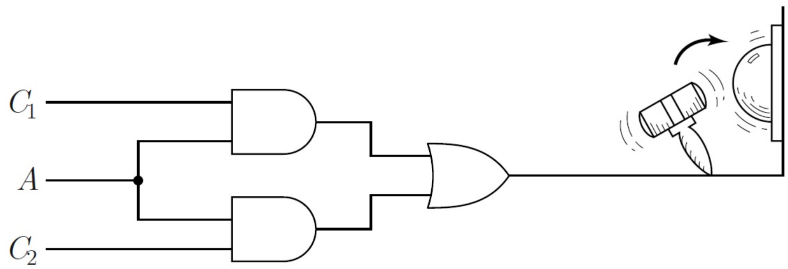
With this available, we can simplify the solution to Example 1.36 to a single gate.

Example 1.37. A zoo has two special cages, each with its own power source, for endangered species. An alarm will be installed in the zoo office that will clang if either (or both) of the special cages are opened, assuming that the cage's power source and the alarm's power source are on. Give a diagram of an appropriate circuit, starting with the three power sources, that would allow the alarm to sound.

Solution. Let A, C_1, and C_2 denote the power supplies for the alarm and the two cages, respectively. For the alarm to sound, we need either A and C_1 to be "on" (that is, $A \wedge C_1$ is T), or we need A and C_2 to be "on" (that is, $A \wedge C_2$ is T). So we need $(A \wedge C_1) \vee (A \wedge C_2)$ to be T. We can produce $A \wedge C_1$ using an AND gate, and we can produce $A \wedge C_2$ using an AND gate; since both require A as an input we need to split the wire associated with A. To get $(A \wedge C_1) \vee (A \wedge C_2)$, we feed the outputs of the two AND gates into an OR gate. This gives the following circuit diagram.



An alarming situation

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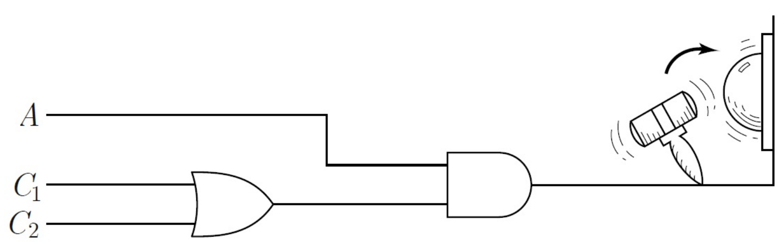
By the Distributive Laws of Section 1.5. Logical Equivalence, we have that $(A \wedge C_1) \vee (A \wedge C_2) \equiv A \wedge (C_1 \vee C_2)$. So we use a simpler circuit (illustrating problem (2) above) using only one OR gate (with C_1 and C_2) and one AND gate (with A and the output of the OR gate) to get the circuit:



Still alarming, but simpler

Figure from page 34 (with corrections)