Mathematical Reasoning, Chapter 1 Study Guide

Chapter 1. Logic.

The following is a *brief* list of topics covered in Chapter 1 of Larry Gerstein's *Introduction to Mathematical Structures and Proofs*, 2nd edition. This list is not meant to be comprehensive, but only gives a list of several important topics.

1.1. Statements, Propositions, and Theorems.

Axioms, laws of logic/deduction.inference/proof, theorem, Bertrand Russell and the *Principia Mathematica*, *a priori* knowledge, mathematical objects are *ideas* and not physically real things, Example 1.1 and the creation of new statements from old under the given rules, propositions, truth values, undecidability, the Continuum Hypothesis, prime number, Fermat number, lemma, corollary.

1.2. Logical Connectives and Truth Tables.

Negation, truth tables, conjunction $P \wedge Q$ and its associated truth values, sentential variables.sentential forms, disjuntion $P \vee Q$ and its associated truth values, exclusive version of or (`xor`) and its truth values.

1.3. Conditional Statements.

Conditional statement, antecedent/hypothesis, consequent/conclusion, $P \Rightarrow Q$ and its truth values, constructing truth tables (as in Example 1.17), converse, biconditional proposition, equivalent propositions, tautology (Exercise 2.3.18).

1.4. Proofs: Structures and Strategies.

Direct proof/*modus ponens*, Aristotle/*Organon*/syllogism, predicate logic, the ideas behind developing a proof (Gerstein's "rambling style"), clean/concise proofs, Kurt Gödel and his incompleteness theorems, the Continuum Hypothesis, the definition of "definition"(!), indirect proof/proof by contradiction/*reduction ad absurdum*, contrapositive, converse.

1.5. Logical Equivalence.

Tautology, logical equivalence, De Morgan's Laws of logic, Replacement Principle, Associative Laws, Commutative Laws, Idempotency Laws, Absorption

Laws, Distribution Laws, Law of Double Negation, Sheffer stroke $\uparrow$ (Exercise 1.5.4).

1.6. Application: A Brief Introduction to Switching Circuits.

Circuits, gates, AND/OR/NOT/XOR gates, using truth tables to convert sentential forms into a switching circuit (see Example 1.36), finding a simplest switching circuit (see Example 1.37).