

Mathematical Reasoning, Chapter 2

Study Guide

Chapter 2. Sets.

The following is a *brief* list of topics covered in Chapter 1 of Larry Gerstein's *Introduction to Mathematical Structures and Proofs*, 2nd edition. This list is not meant to be comprehensive, but only gives a list of several important topics.

2.1. Fundamentals.

Naive set theory versus axiomatic set theory, undefined terms of set and element, membership symbol, equal sets, multi-set, natural numbers, integers, rational numbers, real numbers, Axiom of Separation, empty set \emptyset .

2.2. Russell's Paradox.

Gottlob Frege, Bertrand Russell, Russell's Paradox, Russell and Whitehead's *Principia Mathematica*, Russell's Paradox explained in terms of a barber, a set theoretic version of Russell's Paradox, the resolution of Russell's Paradox and the role of the set of all sets, Example 2.7.

2.3. Quantifiers.

Variable, existential quantifier \exists , universal quantifier \forall , universal set, statements involving quantifiers and negations of such statements, uniqueness existential quantifier.

2.4. Set Inclusion.

Subset, set inclusion \subseteq , proper subset, transitivity of subset inclusion (Theorem 2.15), equality of sets in terms of subset inclusion (Theorem 2.17).

2.5. Union, Intersection, and Complement.

Relative complement of sets $A - B$, union of sets, intersection of sets, Venn diagrams, laws of intersection/union/complement (Theorem 2.16).

2.6. Indexed Sets.

Index set, indexing points in the plane with ordered pairs of real numbers (Example 2.28(c)), union and intersection of an indexed family of sets, elements outside an index union or indexed intersection (Note 2.6.A), laws of intersection/union/complement of an indexed family of sets (Theorem 2.32).

2.7. The Power Set.

Power set, decision tree, vertices/nodes, leaf, graph, root of a tree, properties of power sets (Theorem 2.36).

2.8. Ordered Pairs and Cartesian Products.

Ordered pair/first coordinate/second coordinate (Definition 2.39), equality of ordered pairs (Theorem 2.40), Cartesian product of two sets, Cartesian products with unions and intersections (Theorem 2.45).

2.9. Set Decomposition: Partitions and Relations.

Partition of a set, block of a partition, relation, universal and empty relations, reflexive/symmetric/transitive relation, examples, equivalence relation, equivalence class, equivalence of a partition Π and an equivalence relation \sim (Theorem 2.59 and 2.62), using a graph to illustrate an equivalence relation (Example 2.60; connected components), partition induced by an equivalence relation, equivalence relation induced by a partition.

2.10. Mathematical Induction and Recursion.

Well-Ordering Principle, The Principle of Mathematical Induction (Weak and Strong versions), basis step, induction, step, induction hypothesis, the number of elements in a power set, prime number, Fibonacci sequence, summation symbol, product symbol, factorial symbol.