

# Mathematical Reasoning, Chapter 3

## Study Guide

### Chapter 3. Functions.

The following is a *brief* list of topics covered in Chapter 3 of Larry Gerstein's *Introduction to Mathematical Structures and Proofs*, 2nd edition. This list is not meant to be comprehensive, but only gives a list of several important topics.

#### **3.1. Definitions and Examples.**

Function between two sets, domain, codomain, image, preimage, a function transforms  $x$  to  $y$ , a function operates on its domain, mapping, transformation, inclusion mapping, well-defined function, equal functions, restriction of a function to a subset of the domain, extension of a function to a superset of the domain (Definition 3.6).

#### **3.2. Surjections, Injections, Bijections, Sequences.**

Range or image of a function, surjection/onto, injection/one-to-one, the “marriage problem,” system of distinct representatives, Hall’s Marriage Theorem, greedy algorithms, bijection/one-to-one correspondence, finite sequence/infinite sequence/sequence, length of a finite sequence,  $n$ -tuple, coordinate, a bijection from  $\mathbb{N}$  to  $\mathbb{N} \times \mathbb{N}$  (Example 3.18), an uncountable set (Example 3.19), Cantor diagonalization argument.

#### **3.3. Composition of Functions.**

Composition of two functions, projections (Example 3.21), Associativity Law of Function Composition (Theorem 3.23), compositions of injections/surjections/bijections (Theorem 3.24), a surjection from  $\mathbb{N}$  to  $\mathbb{Q}^+$  (Example 3.15), inverse function of a bijection, the inverse of a bijection is also a bijection, equivalent conditions for a function to have an inverse (Theorem 3.27), Cancellation Laws (Corollary 3.28), the inverse of a composition (Theorem 3.29).