Mathematical Reasoning, Chapter 4 Study Guide

Chapter 4. Finite and Infinite Sets.

The following is a *brief* list of topics covered in Chapter 4 of Larry Gerstein's *Introduction to Mathematical Structures and Proofs*, 2nd edition. This list is not meant to be comprehensive, but only gives a list of several important topics.

4.1. Cardinality; Fundamental Counting Principles.

Hilbert hotel, same cardinality sets, equipotent sets, one-to-one correspondence, properties of equipotence $\approx$ (Theorem 4.2), equipotence of open intervals of real numbers and $\mathbb{R}$ (Example 4.5), finite set, infinite set, properties of finite sets (Theorem 4.8), The Pigeonhole Principle (Corollary 4.9), properties of finite and infinite sets (Theorems 4.11, 4.12, and 4.13), Addition Rule (Theorem 4.14), pairwise disjoint sets and Corollary 4.15, the Inclusion Exclusion Principle and Corollary 4.16, cardinalities of Cartesian products of finite sets (Theorem 4.17 and Corollary 4.18), Multiplication Rule and Theorem 4.21, applications of the Multiplication Rule (Example 4.23).

4.2. Comparing Sets, Finite or Infinite.

Equivalence of cardinality relations and injections/bijections for finite sets (Theorem 4.26), definition of equalities and inequalities for cardinalities of infinite sets (Definition 4.27), Schröder-Bernstein Theorem (Theorem 4.28), $[0,1] \approx (0,1)$ (Example 4.29), implications of cardinality inequalities for all sets (Theorem 4.30), $\#S < \#P(S)$ for all sets S (Cantor's Theorem I, Theorem 4.31) and the implication that there is no largest set, Georg Cantor.

4.3. Countable and Uncountable Sets.

Countably infinite sets, countable set, uncountable set, $\mathbb{Z}$ is countably infinite (Example 4.34), properties of countable sets (Theorems 4.35, 4.36, 4.37, and 4.39), $\mathbb{Q}$ is countable (Theorem 4.40), $\mathbb{R}$ is not countable (Cantor's Theorem II, Theorem 4.41), Cantor's diagonalization argument, the set of irrational numbers is uncountable (Corollary 4.42), an application of countable and uncountable sets (Example 4.43).

4.4. More on Infinity.

Richard Dedekind's and Charles Sanders Peirce's definition of an infinite set (Theorem 4.44).

4.5. Languages and Finite Automata.

Alphabet Σ, word, length of a word, empty word, the set of all words Σ^*, Σ^* is countably infinite, language L, uncountably many languages (Theorem 4.51), finite automaton (Definition 4.52), initial state, state, transition function δ, examples of finite automata (Example 4.53), transition diagram, vertices, arcs, accept a word (Definition 4.57), examples of acceptance of words (Example 4.58), regular language (Definition 4.59), Kleene's Theorem, concatenation of words, Plumping Lemma (Lemma 4.60), example of a non-regular infinite language (Example 4.61(a)).