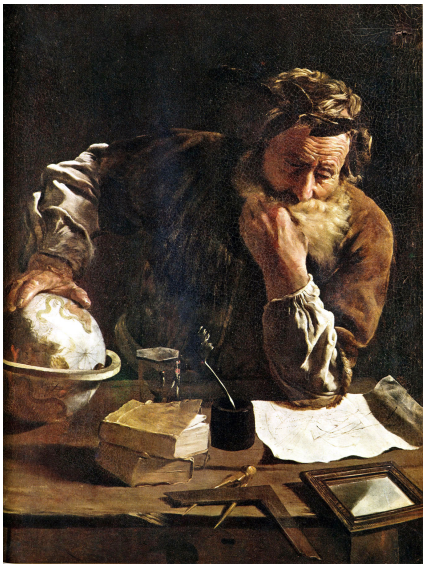
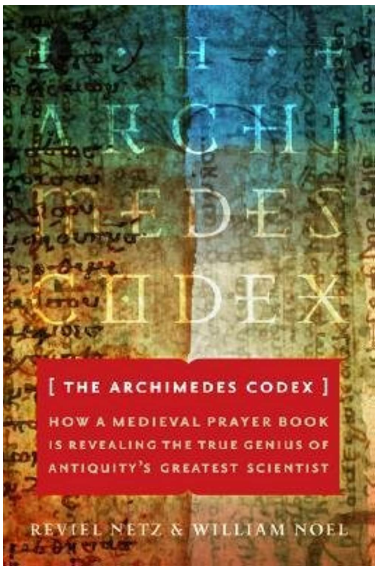
Supplement. Archimedes' *Method*, Part 1

Note. In this supplement, we explore a work by Archimedes of Syracuse (287 BCE–212 BCE) which anticipated the development of integral calculus 1900 years before before Isaac Newton (January 4, 1643–March 31, 1727) and Gottfried Wilhelm Leibniz (July 1, 1646–November 14, 1716). The work is Archimedes *Method*. We explore its content and the interesting history of how we came to know about this content through a single surviving copy. Most of the content of this supplement is based on Reviel Netz and William Noel's *The Archimedes Codex: How a Medieval Prayer Book is Revealing the True Genius of Antiquity's Greatest Scientist* (Da Capo Press, 2007). This book describes work done after the printing of the latest edition of Howard Eve's *An Introduction to the History of Mathematics* (6th edition, Saunders Publishing, 1990), and so is included in these notes as a supplement to the History of Mathematics (MATH 3040) class notes on Section 6.2. Archimedes.





An imagined appearance of Archimedes in the painting *Archimedes Thoughtful* by Dominico Fetti in 1620 (left), from the Wikipedia page on Archimedes (accessed 10/2/2023)

Note AM.A. The most common reference in English for Archimedes' original works is Thomas Heath's *The Works of Archimedes* (Cambridge University Press, 1897) (this is still in print and available from Dover Publications). However, Heath's book excludes the *Method*, which was not rediscovered until 1906 (though the Dover version includes a 1912 English translation by Heath). However, it is commented in *The Archimedes Codex* that (see pages 30 and 31):

> "For Archimedes [in English] there existed only T. L. Heath's poor paraphrase published in 1897, which simply ignores Archimedes' mathematical language. ... If you open an edition from that era, the diagrams you find are not based upon what is actually drawn in the original manuscripts. The diagrams represent, instead, the editor's own drawing. I [Will Noel] was shocked to realize this and began to consider whether I should produce, for the first time, an edition of the diagrams."

We start our history of Archimedes *Method* with the story of its modern-day rediscovery.

Note AM.B. While creating a catalogue of manuscript holdings of the Metochion of the Holy Sepulcher in Constantinople (today, Istanbul, Turkey) in 1899, it was recorded that manuscript number 355 was a prayer book which was written over the erased text of another work. The catalog described a section of the faintly-visible erased text, which was of some mathematical content. This entry was brought to the attention of Johan Heiberg (see Note 5.3.J of Section 5.3. Euclid's "Elements"), who was working in Copenhagen, Denmark. Hieberg recognized the described work

as that of Archimedes. Heiberg tried to have the manuscript sent to him, but when this failed he took his 1906 summer vacation in Constantinople. The librarian at the Metochion allowed him to study the manuscript. Heiberg had found, under the writing of the prayer book, an unknown manuscript of Archimedes. It appears in the form of a letter from Archimedes to Eratosthenes (276 BCE–194 BCE; see Section 6.3. Eratosthenes). Heiberg published his findings in 1907 in "Eine Neue Archimedeshandschrift" ["A New Archimedes Manuscript," with a German introduction and the *Method* in Greek], *Hermes*, **42**(2), 235–303 (1907) (available online JSTOR; accessed 10/2/2023). Heiberg worked with a mathematical colleague, H. G. Zeuthen, to help him reconstruct the diagrams from Archimedes text. Heiberg and Zeuthen published a German translation of the Greek version of the *Method* in "Eine neue Schrift des Archimedes" ["A new manuscript by Archimedes"], *Bibliotheca Mathematica* **7**(3), 321–363 (1906-07); this is available on archive.org (accessed 10/2/2023). Heath's English translation of the *Method* is based on Heiberg's two 1907 translations.

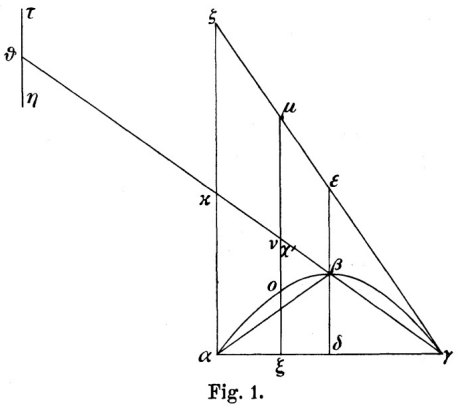


Fig. 1.


Hermes XLII. Bd. 1907. S. 235.


Eine neue Schrift des Archimedes.

Von J. L. Heiberg und H. G. Zeuthen in Kopenhagen.

Figure 1 and the first page of Heiberg's 1907 *Hermes* paper with an image of the *Method*, and the first page of Heiberg and Zeuthen's 1907 in German.

Note AM.C. The classical works of the Greek geometers give no information of the motivation or thinking process in producing their results. As Heath says in his *The Works of Archimedes* (see page 7 of his supplement on the *Method*): "... they seem to have taken pains to clear away all traces of the machinery used and all the litter, so to speak, resulting from tentative efforts, before they permitted themselves to publish ... the results obtained." In fact, this is a common approach to presenting mathematical research publications even today (and can be found, regrettably, in some math textbooks). The *Method* is, to an extent, an exception. Archimedes describes his method of discovery of certain theorems concerning areas and volumes. In so doing, he is careful to distinguish between the intuitive means by which he *explores* the validity of the results, and the rigorous arguments using geometrical methods which establish the theorems (that is, the *proofs*).

Note AM.D. We mentioned in Note AM.B that the *Method* appears as faintly written erased text in a prayer book. The work is an example of a *palimpsest* (more on this in the next note). Like most medieval manuscripts, it is written on parchment. Parchment is the skin of an animal that that has been scraped and prepared for use as a writing surface. It is durable, flexible, and expensive. Once written on, it is possible to scrape off the writing, clean the parchment, and then reuse it for a second round of writing. The writing of the *Method* is written in a script consistent with that used in the third quarter of the tenth century (i.e., 950 CE–975 CE). Notice that this date puts the manuscript more closely to our time than to the time of Archimedes! On page 84 of *Archimedes' Codex* it is stated:

> "It is certainly possible that [the scribe] copied a sixth-century manuscript, but the evidence is not conclusive. This is perhaps the most important unanswered question about our manuscript."

The folios of the manuscript (that is, the pages; the term "folio" refers to the front and back together so that the number of folio of a manuscript is twice the number of what we would call "pages") measured 12 inches by 7.5 inches, the writing appears in two columns, and the written area is 9.5 inches by 5.5 inches (so there are generous margins). The lack of marginal comments indicates that it was not read very often. The manuscript, called "Codex C," is the only known source for Archimedes' *Method* and *Stomachion*, and the only source in Greek of *On Floating Bodies*. It is the earliest surviving text of Archimedes work.

Note AM.E. In the thirteenth century, another scribe sits down at his table to make a copy of a prayer book. He has a pile of parchment that is familiar to us. The parchment, in part, consisted of several of the folios of Codex C of Archimedes. The scribe was in the process of making a palimpsest. The Greek word *palimpsestos* means "scraped again." A palimpsest is made by removing the parchment from an existing manuscript, scraping it clean of the old writing (an option with the durable parchment material), separating the folios that were joined in the margin of the manuscript, rotating them 90°, writing on the clean surface, and bounding the new folios (which are now half the size of the original folios). See the figure below. This process results in the jumbling of the order of the original codex, the loss of the material in the center of each folio of the original codex in the gutter of the new palimpsest, and, of course, the fact that the Codex C material has been

scrubbed off and written over.

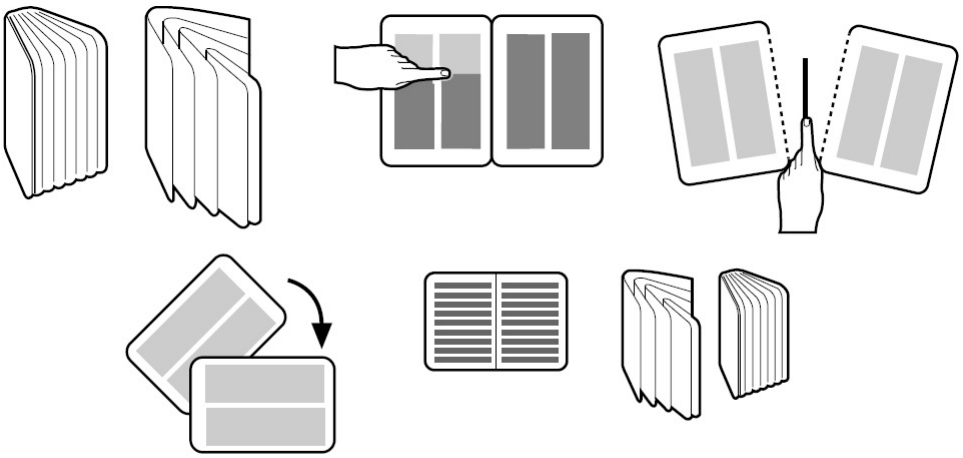


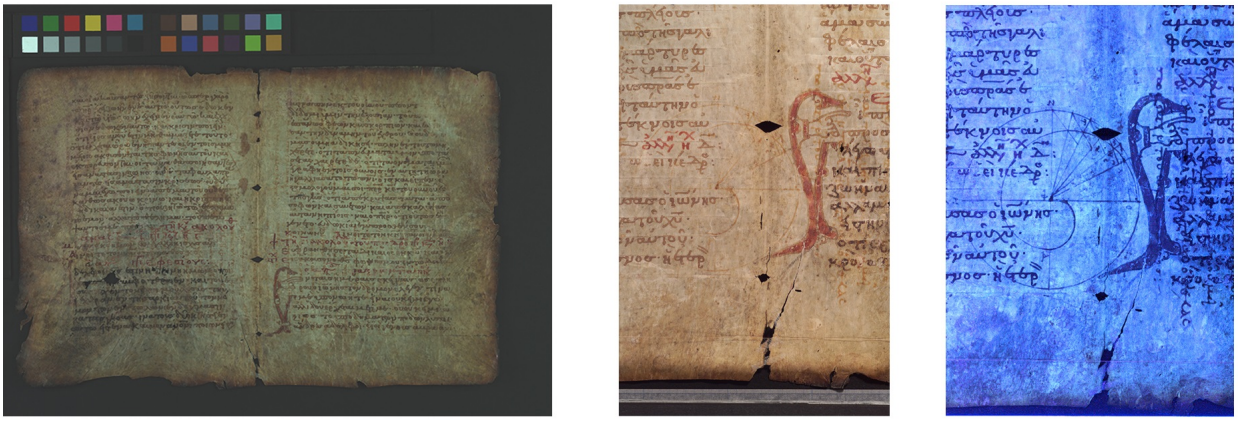
FIGURE 5.1 *How to make a palimpsest*

From page 123 of *The Archimedes Codex*

The loss of information is reflected in Heiberg's work. For the two or three hidden lines of Archimedes in the gutter of the prayer book, Heiberg makes a guess as to what was written (this can also be seen in Heath's English version in the Dover Publications version of his *The Works of Archimedes*). However, working in 1906 Heiberg was limited in the technology available to him. Today, ultraviolet light would be used to increase the contrast and bring out hidden details. Heiberg simply used a magnifying glass to inspect the original, but he also took 65 photographs. These photographs (found in the Royal Library of Denmark in 2000) give a record of what the prayer book looked like in 1906. One of his photographs is given above in Note AM.B, where you can see the vertical faint writing under the horizontal writing of the prayer book. As we'll see, the book had some bad years ahead of it after Heiberg studied it. To give a quick idea of the content of the prayer book, we quote again *The Archimedes Codex* (pages 124 and 125):

> "The first piece of parchment in his new codex [the prayer book] contained *On Floating Bodies*. He [the scribe] covered it with a blessing for loaves at Easter. . . . He wrote over over the beginning of the *Method* with a prayer of marriage. Over a later section of the *Method*, he wrote a prayer recited at the foundation of a church. And, note this, over Archimedes' critical proposition 14, he wrote a prayer for the dead."

The following image, from Frank J. Swetz, "Mathematical Treasure: The Archimedes Palimpsest," *Convergence* (July 2013), gives a photograph of a double folio from the prayer book (left), a close up of the gutter in which a faint drawing of a spiral can be seen (center), and a photograph made with ultraviolet fluorescent light that shows the spiral more clearly (right).



Note AM.F. The *Method* is written in the form of a letter from Archimedes to Eratosthenes (276 BCE–194 BCE) and was composed sometime before the Second Punic War (a war between Rome and Carthage [located in North Africa] fought between 218 BCE and 201 BCE). It starts:

> "Archimedes to Eratosthenes: greetings! Since I know you are diligent, an excellent teacher of philosophy, and greatly interested in any mathematical investigation that may come your way, I thought it might be appropriate to write down and set forth for you a certain special method. . . I presume there will be some among the present as well as future generations who by means of the method here explained will be enabled to find other theorems which have not yet fallen to our share."

The letter would have been written on papyrus which was wrapped around a wooden core to form a "roll." There was virtually no punctuation and no spaces were left between words. It was sent to Alexandria, where Eratosthenes was. There, Heron of Alexandria (circa 10 CE–75 CE) read the work (we'll see Heron in more detail in Section 6.6. Heron). This is known because Heron quotes the *Method* in his *Metrica*, which itself survives in only one manuscript. For centuries before and after Heron's reference, no other mention of the *Method* is known. This note is based on pages 66, 67, and 70 of *The Archimedes Codex*.

Note AM.G. Few of the fragile papyrus rolls survive. If Archimedes' *Method* had not been copied onto more durable material, it probably would not have survived. For example, Pappus of Alexandria (circa 290 CE–350 CE) mentions a treatise by Archimedes on semi-regular polyhedra, which is known from no other references and which did not survive (more on Pappus is given in Section 6.9. Pappus). Much of the work of Archimedes was so deep and complicated that few people could understand the arguments, leading to a low demand for copies of it. However, some of the information survived the twenty-plus centuries following Archimedes'

time. Eutocius of Ascalon (circa 480 BCE–540 BCE) did more than anyone else to ensure the survival of Archimedes' work during the 6th century. He read, studied, and wrote commentaries on the works. He wrote commentaries on *Sphere and Cylinder* (in two parts, I and II), *Measurement of the Circle*, and *Balancing Planes* (more on Eutocius is given in Section 6.10. The Commentators). Eutocius' treatises survives. It seems that Eutocius also prepared an edition of several of Archimedes' works, together with his commentaries. It would have been written on parchment and bound within wooden covers. That is, it was in the newest form of data storage of the time, a *codex* (or book). Whereas a papyrus roll would eventual decay into dust, a codex could survive for centuries. This note is based on pages 72–74 of *The Archimedes Codex*.

Note AM.H. There may have been many copies of the *Method* outside of Alexandria in the third century CE (though we will likely never know), but by the end of the sixth century there were hardly any. The great cities of the ancient Mediterranean world were raided during this time. Rome was sacked by the Goths (Germanic peoples) in 410 CE, Antioch (in southern Turkey, where modern-day Atakya is located) was sacked by the Persians in 540 CE, and Athens was sacked by the Slavs (of east-central Europe) in 580 CE. We discussed the demise of Alexandria itself in Section 5.1. Alexandria and (with more attention on the Mouseion and Library of Alexandria) in Section 6.1. Historical Setting. We quote *The Archimedes Codex* (pages 74 and 75) for a description of the events at the time:

> "As the ancient world disappeared, its gods went with it. And as Christianity became the official religion of the Roman Empire, many classical texts, if they were not condemned as dangerous, were dismissed as irrelevant. It is not that Christians willfully destroyed them very often; they just ceased to copy them. Scribes put their energies into Christian texts. . . . Fewer people than every before had the resources to read [Archimedes], and even fewer would have read him, if they could."

In 6.1. Historical Setting, we noted that Roman emperor Constantine (February 27 circa 272 CE–May 22 337 CE) converted to Christianity around 312 CE, and built up the city of Constantinople on the site of the ancient city of Byzantium (the site of modern-day Istanbul, Turkey); see Note 6.1.D. Constantinople would become a center of learning. The emphasis was on Christian works and Constantine ordered that fifty copies of the Bible to be written (with whatever content was thought appropriate at that time). But the classics were also copied and collected. A proposal for a scriptorium (i.e., a writing center) for the writing of new copies of the classics, which would make Constantinople a center of culture. The proposal may have been instituted. Records indicate that scribes conversant in Greek and Latin were hired to make transcriptions and repairs of books. Eastern Roman Emperor Justinian (May 11, 482 CE–November 14, 565 CE) oversaw the construction of the Church of Hagia Sophia in Constantinople between 532 CE and 537 CE (in the 15th and 16th century it was converted to a mosque, and it still stands today; see the image below). Two architects of this church were Anthemius of Tralles (474 CE–534 CE) and Isidore of Miletus (442 CE–537 CE). In addition to being architects, both were also mathematicians. Anthemius wrote a book *On Burning Mirrors* which described the focal properties of a parabola, and a book *On remarkable*

mechanical devises which was a survey of configurations of mirrors. Isidore wrote a commentary on Heron's treatise *Vaulting* (we consider Heron (circa 10 CE–circa 75 CE) in Section 6.6. Heron). A student of Isidore's (whose name is unknown, but he acknowledged Isidore in the work) made a copy of Eutocius' commentary on *Sphere and Cylinder I* (see Note AM.G above).

The Hagia Sophia in Constantinople. Image from the History.com website (accessed 10/7/2023)

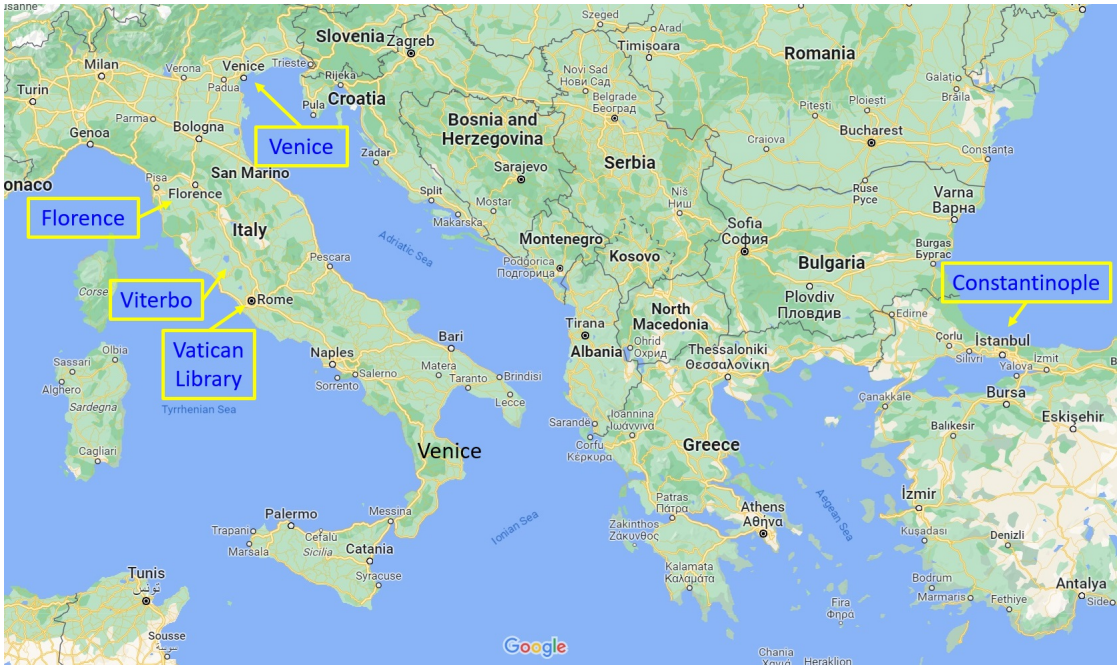
The presence of Archimedes' works in Constantinople was instrumental in the transmission the work to our time. For three hundred years after the time of Isidore, Archimedes' works disappear from recorded history. However, during this time his works were stored away in Constantinople. The city was protected by massive walls, and it was the only consequential city of the ancient Mediterranean world to survive the Early Middle Ages (or the "dark ages") unmolested. Its survival during this time provided a sanctuary for much ancient literature, including that of Archimedes. This note is based on pages 74–77 of *The Archimedes Codex*.

Note AM.I. Eastern Roman Emperor (or "Byzantine Emperor") Nicephorus (or "Nikephoros") invaded Bulgaria and brutally attacked cities and civilians. He was killed in 811 in the Battle of Pliska in the Balkan Mountains. For the next few decades (and several emperors), things didn't look politically good for Constantinople. In 867, Basil I assassinated emperor Michael II (known as "The Drunkard") and under Basil I Constantinople became the capital of the greatest empire in the Mediterranean world at the time. The ninth and tenth centuries mark the "Byzantine Renaissance," during which there were spates of building and creation of art. Scholars started reading and copying the classic works that had been safely stored in their libraries. Ninth century scribes copied texts in a script different from that used In Isidore's day. Before the ninth century, scribes used only capital letters called "majiscules." After the ninth century, scribes used a script whose letters could be joined up and took up less space; this is called "miniscule." Once scribes copied the the old majiscule texts into the new miniscule script, the old texts were no longer necessary and were discarded. This resulted in the survival of few of the fifth and sixth century majiscule manuscripts. The survival of nearly all of the ancient Greek authors depended on the miniscule manuscripts copied in Constantinople. This is also the case for the works of Archimedes. Three physical objects containing work of Archimedes were preserved by this sequence of events in Constantinople. They are Codex A, Codex B, and Codex C. Codex C is the only one to survive, though Codices A and B are known by copies and translations that have reached our time. Archimedes' work was in the codices as follows:

Codex A: *Balancing Planes*, *Quadrature of the Parabola*, *Sphere and Cylinder*, *Measurement of the Circle*, *Spiral Lines*, *Conoids and Spheroids*, *Sand Reckoner*;

Codex B: *Balancing Planes*, *Quadrature of the Parabola*, *On Floating Bodies*;
Codex C: *Balancing Planes*, *Sphere and Cylinder*, *Measurement of the Circle*, *Spiral Lines*, *On Floating Bodies*, *Method*, *Stomachion*.
Codex C (dating from 950 CE–975 CE, as observed in Note AM.D) is now the oldest surviving manuscript of Archimedes' work in Greek by over four hundred years. This note is based on pages 77–81 of *The Archimedes Codex*.

Note AM.J. We now give the sad part of the story of the codices. In 1204 the Fourth Crusade (which occurred under the reign of Pope Innocent III) started with with crusaders provided by the head of state of Venice for a certain price. However, complete funds were not available and the crusaders struck a deal with Alexius Anelus (grandson of the ousted Emperor of Constantinople, Isaac II) to install him on the throne of Constantinople for a generous fee. The crusaders succeeded, but Alexius was unable to produce the promised fee. While waiting to be paid, the Christian crusaders attacked a mosque. In the attack, a fire broke out and spread. Hundreds died and a three mile wide area was burned through the center of Constantinople. The citizens of Constantinople had enough and strangled Alexius, ending his reign. Hostilities began again between the crusaders and the people of Constantinople. The people surrendered on April 13, 1204. The crusaders took over the city, the debt to Venice was paid off, and the Catholic faith was imposed. Many of the classic works that had been safe in Constantinople, were burned. This was the end of Constantinople as a haven for the classics, and any surviving works of Archimedes would be found elsewhere. This note is based on pages 117–119 of *The Archimedes Codex*.



Based on an image from

the Google Maps (accessed 10/25/2023)

Note AM.K. In 1881, a manuscript was found in the Vatican Library. It was a translation of the works of Archimedes from the Greek to Latin. It was written in 1269 in Viterbo, Italy. A 1311 catalogue of manuscripts in Viterbo which belonged to the Pope indicates that there were two manuscripts of Archimedes' work available to the translator. These two works are what is described above in Note AM.I as Codex A and Codex B. Unfortunately, the 1311 catalogue is the last known reference to the Greek version of Codex B. Codex A is known to have been in the possession of Pope Nicholas V in 1450, since he commissioned a translation of it. In 1492, Codex A was found in the library of Giogio Valla in Venice, and a copy was made. This copy survives and in the Laurentian Library in Florence, Italy. Giorgio Valla's library was bought by Alberto Pio who, upon his death in 1531, passed Codex A to his nephew Ridolfo Pio. Ridolfo died in 1564 and this marks

the disappearance of Codex A. Though now lost, Codices A and B are responsible (primarily through the translations mentioned here) for transmitting the ideas of Archimedes to the thinkers of the Renaissance, and then on the the modern world. The first printed version of Archimedes work appeared in 1544. This work would be available to Leonardo da Vinci (April 15, 1452–May 2, 1519), Galileo (February 15, 1564–January 8, 1642), Newton (January 4, 1643–March 31, 1727) and had, no doubt, an impact on the birth of modern science. Leonardo in his notebooks reveals a knowledge of several of the works of Archimedes. In particular, Leonardo was interested in *Balancing Planes* in which Archimedes shows how to find the center of gravity of a plane figure. Leonardo went beyond this, considered centers of gravity of solid objects, and gave a theorem to find the center of gravity of a tetrahedron. However Archimedes had also found centers of gravity of solids, including some with curved surfaces. This was now known to Leonardo because it appeared in Archimedes' *Method* in Codex C. As we saw in Note AM.E, Codex C disappeared from circulation in the thirteenth century when it was turned into a prayer book in the form of a palimpsest. This note is based on pages 119–122 of *The Archimedes Codex*.

Note AM.L. The Monastery of Saint Sabas is located about 5 miles east of Bethlehem, half way between Bethlehem and the Dead Sea in the West Bank Palestinian territory. In 1834 the library at Saint Sabas contained over 1,000 manuscripts. Somehow, sometime the Archimedes palimpsest had made its way to this Saint Sabas and it was one of the manuscripts in the library. It is known to have been there because it was described as part of the Saint Sabas collection by a Greek

scholar in 1899. As a prayer book, the palimpsest showed signs of use. Its edges were charred, many of the folios were covered with drops of wax (since it would have been read by candlelight), and marginal additions were made in places. Sadly, about one third of the codex (or 60 folios) were missing. The underlying work of Archimedes was ignored be the readers at the monastery. "Abstract mathematics is not a priority at St. Sabas. Archimedes was effectively buried at the monastery for at least three hundred years" [*Archimedes Codex*, page 129]. This note is based on pages 128–129 of *The Archimedes Codex*.

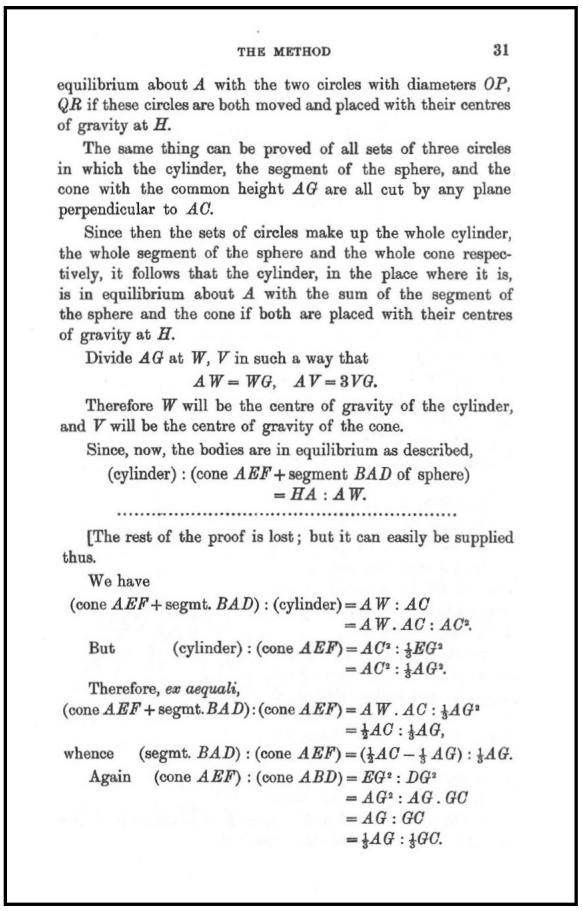
Note AM.M. In 1876, the Cambridge University Library bought 44 fragments of manuscripts from the estate of biblical scholar Constantin Tischendorf (January 18, 1815–December 7, 1874). Tischendorf is best known for discovering the oldest surviving complete copy of the New Testament, known today as the "Codex Sinaiticus." In 1846, he published *Travels in the East* in which he describes a visit to the Metochion of the Holy Sepulcher in Constantinople (which we mentioned in Note AM.B in connection with the work of Heiberg). Tischendorf described a palimpsest there that contained mathematics. One of the folio's in the collection of the Cambridge Library is a page from the palimpsest containing material from *Sphere and Cylinder*, which fits between folios 2 and 3 of the palimpsest; see the figure below. The only conclusion is that Tischendorf had removed the folio from the palimpsest during his visit to Constantinople and added it to his collection (*not* an accepted scholarly behavior!). The palimpsest was in the catalogue of the Metochion in Constantinople in 1899. What isn't known, is how or when it was moved from Saint Sabas back to Constantinople. In the early 19th century, the

manuscripts of Saint Sabas were incorporated into the property of the Greek (Eastern Orthodox) Church, so it is plausible that this is related to the movement of the manuscripts and this lead to prayer book making its way back to Constantinople, where it was originally created. This note is based on pages 130–131 of *The Archimedes Codex*.

Front and back of the "Tischendorf page," from the University of Cambridge Digital Library (accessed 10/26/2023)

Note AM.N. In Note AM.B, we saw that Johan Heiberg visited the Metochion of the Holy Sepulcher in Constantinople in 1906, studied and photographed the palimpsest, and published his results in 1907, which Heath translated into English in 1912. Now that we know the physical construction of the plampsest, we see why there is missing information in both Heiberg and Heath's versions (see the figure below). Recall that Heiberg had worked with H. G. Zeuthen to create diagrams

based on Heiberg's version of the Archimedes text. So, after Heiberg's and Heath's publications, the diagrams of Archimedes still had not been studied. Interest was great in additional study of the palimpsest.



THE METHOD 31

equilibrium about A with the two circles with diameters OP, QR if these circles are both moved and placed with their centres of gravity at H.

The same thing can be proved of all sets of three circles in which the cylinder, the segment of the sphere, and the cone with the common height AG are all cut by any plane perpendicular to AC.

Since then the sets of circles make up the whole cylinder, the whole segment of the sphere and the whole cone respectively, it follows that the cylinder, in the place where it is, is in equilibrium about A with the sum of the segment of the sphere and the cone if both are placed with their centres of gravity at H.

Divide AG at W, V in such a way that

$$AW = WG, \quad AV = 3VG.$$

Therefore W will be the centre of gravity of the cylinder, and V will be the centre of gravity of the cone.

Since, now, the bodies are in equilibrium as described,

$$\text{(cylinder)} : \text{(cone } AEF + \text{segment } BAD \text{ of sphere)} = HA : AW.$$

.....

[The rest of the proof is lost; but it can easily be supplied thus.

We have

$$\begin{aligned} (\text{cone } AEF + \text{segmt. } BAD) : (\text{cylinder}) &= AW : AC \\ &= AW \cdot AC : AC^2. \end{aligned}$$

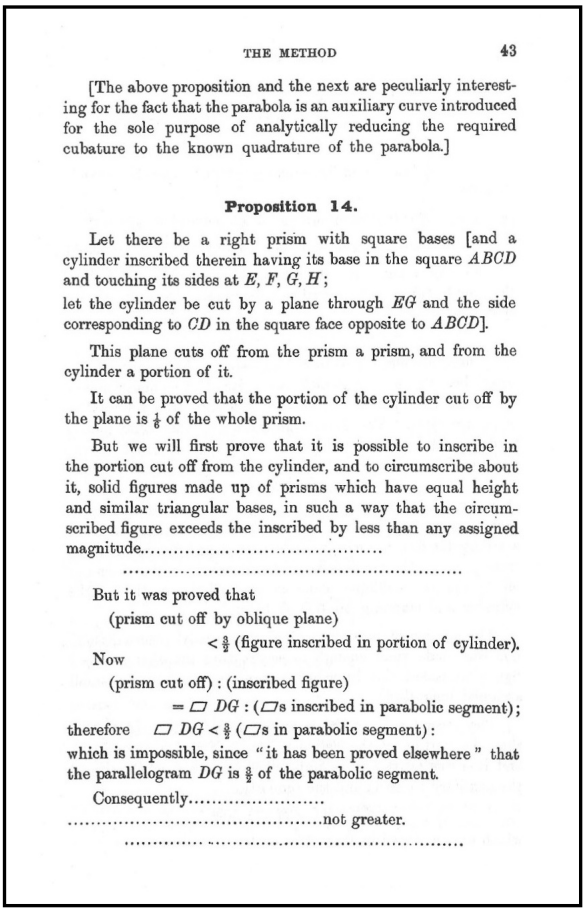
But
$$\begin{aligned} (\text{cylinder}) : (\text{cone } AEF) &= AC^2 : \tfrac{1}{3}EG^2 \\ &= AC^2 : \tfrac{1}{3}AG^2. \end{aligned}$$

Therefore, *ex aequali*,

$$\begin{aligned} (\text{cone } AEF + \text{segmt. } BAD) : (\text{cone } AEF) &= AW \cdot AC : \tfrac{1}{3}AG^2 \\ &= \tfrac{1}{3}AC : \tfrac{1}{3}AG, \end{aligned}$$

whence
$$(\text{segmt. } BAD) : (\text{cone } AEF) = (\tfrac{1}{3}AC - \tfrac{1}{3}AG) : \tfrac{1}{3}AG.$$

Again
$$\begin{aligned} (\text{cone } AEF) : (\text{cone } ABD) &= EG^2 : DG^2 \\ &= AG^2 : AG \cdot GC \\ &= AG : GC \\ &= \tfrac{1}{3}AG : \tfrac{1}{3}GC. \end{aligned}$$



THE METHOD 43

[The above proposition and the next are peculiarly interesting for the fact that the parabola is an auxiliary curve introduced for the sole purpose of analytically reducing the required cubature to the known quadrature of the parabola.]

Proposition 14.

Let there be a right prism with square bases [and a cylinder inscribed therein having its base in the square $ABCD$ and touching its sides at E, F, G, H;

let the cylinder be cut by a plane through EG and the side corresponding to CD in the square face opposite to $ABCD$].

This plane cuts off from the prism a prism, and from the cylinder a portion of it.

It can be proved that the portion of the cylinder cut off by the plane is $\frac{1}{6}$ of the whole prism.

But we will first prove that it is possible to inscribe in the portion cut off from the cylinder, and to circumscribe about it, solid figures made up of prisms which have equal height and similar triangular bases, in such a way that the circumscribed figure exceeds the inscribed by less than any assigned magnitude.....

.....

But it was proved that

(prism cut off by oblique plane) $< \frac{8}{9}$ (figure inscribed in portion of cylinder).

Now

(prism cut off) : (inscribed figure) $= \square\, DG$: ($\square$s inscribed in parabolic segment);

therefore $\square\, DG < \frac{8}{9}$ ($\square$s in parabolic segment):

which is impossible, since "it has been proved elsewhere" that the parallelogram DG is $\frac{8}{9}$ of the parabolic segment.

Consequently.....

.....not greater.

....

Pages 31 and 43 of Heath's English translation of Archimedes' *The Method*, showing "·····" where the work of Archimedes vanishes into the binding of the palimpsest or is otherwise obscure. On page 31, Heath/Heiberg speculate as to the precise structure of the missing work.

However, it could not be found. By 1938, the Metochion manuscripts had been moved to the National Library of Greece (in Athens). Around the end of World War I, Constantinople entered a period of political instability and sporadic violence

(some of it aimed at the Greek Church). So the relocation of the manuscripts (of which there are no records) was a fortunate move. However, a number of the manuscripts did not make it to Greece. The Archimedes palimpsest is one of the works that disappeared. By 1947, it was in the possession of Anne Guersan of Paris, who was given the palimpsest by her father Marie Louis Sirieix (who died in 1956). In the 1960s and 1970s Anne investigated the document and by 1970, she new the importance of what she had. She tried to sell it privately to an individual or institution in the 1970s, but was unsuccessful. It had suffered from mold damage, was fragile and decayed, and now had four illuminated portraits added (see the image below). By comparing the current condition of the palimpsest to Heiberg's photographs of 1906, it can be seen that the much of damage and the addition of the (forged) portraits occurred in the 20th century (probably between 1940 and 1960). The portraits were copied (in part, by tracing) from a 1929 publication of Greek manuscripts in the Bibliothèque National in Paris (these same portraits were fraudulently added to a different manuscript, also from the Metochion, in the holdings of the Duke University Library, streamlining the identification of them as such in the Archimedes palimpsest). Chemical analysis of the paintings reveal the use of a paint ("phthalocyanine green") that was commercially available until 1938 (giving a definitive earliest possible date for the addition of the forgeries). A letter dated February 10, 1934 from Salomon Guerson, owner of an Parisian antiquities business, surfaced indicating that the manuscript was known to include the work of Archimedes and stating that Guerson was trying to sell it for $6,000 (it is clear from the letter that Guerson saw the value of the manuscript was due to the presence of Archimedes and not in the fact that it is a prayer book). The

specifics of how the palimpsest worked its way from Constantinople to Paris is still unknown. At some point, possibly in the early years of the second world war, Marie Sirieix acquires the manuscript from Guerson. In *The Archimedes Codex*, it is speculated that the German invasion of Paris in June of 1940 (and the fact that Guerson was Jewish and struggling to survive at the time) lead to a situation where Guerson was responsible for the added forgeries, and that he sold the manuscript to Sirieix (or so is speculated). Sirieix then stored the manuscript (possibly in a damp setting, leading to the mold damage) and then returned to fighting in the French resistance. This note is based on pages 133–135 and 167–172 of *The Archimedes Codex*.

Image of the Saint John forgery on folio 57r,
from The Huntington Library website (accessed 10/27/2023).

Note AM.O. On October 29, 1998 the palimpsest came up for auction through Christie's auction house in New York City. Christie's prepared a catalogue for the palimpsest describing it, but not recording the owner's name (though it was Anne Guersan of Note AM.N). A few days earlier, the Greek Orthodox Patriarchate of Jerusalem took out a restraining order to stop the auction, claiming the book was the property of the church. The reserve price was set at $800,000. The Minister of Culture of Greece had set up a consortium to raise funds to bring the it back to Greece, and was represented by a bidder. Simon Finch, a book dealer from London, was bidding for an anonymous person (he would simply say that the person was American and "not Bill Gates"). These were the only two bidders. The palimpsest sold for $2,000,000 (plus 10% commission for Christie's); it was won by the anonymous American. Will Noel, curator of the Walters Art Museum in Baltimore, Maryland, sent an email to Finch asking to be granted access to the palimpsest for analysis (thinking it unlikely that such access would be granted). Finch contacted the new owner, known only as "Mr. B," and access was granted. On January 19, 1999 the palimpsest was delivered to the Walters Museum. Noel became the coordinator of a team that thoroughly studied and imaged the fragile, moldy relic of Archimedes work. It needed conservation, imaging, and scholars to study it. The team would take care of this, all funded by Mr. B. The conservation team was lead by Abigail Quandt of the Walters Museum. Though it is an "Art" museum, the Walters is equipped to handle such a conservation project; Quandt had experience working on a number manuscripts, including the Dead Sea Scrolls and the *Book of Kells*. A Request for Proposals was issued for an imaging group. Work begin in 1999, though the "disbinding" (removal of the folios from the bound

palimpsest for closer study) took from April of 2000 to November of 2004. This note is based on pages 2–24 and 179 of *The Archimedes Codex*. We continue the story of the study of the palimpsest and the mathematical results in Part 2 of this supplement, Supplement. Archimedes' *Method*, Part 2.

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