1.2. Number Bases

Note. In this section we give a brief history of various number bases, concentrating on bases 2, 5, 12, 20, and 60. We give an example of each and motivate several of them with body counting (that is, using parts of the body to represent numbers). Of course we use base 10 because of counting with our fingers.

Note. Early number systems involved in arranging numbers into convenient groups. Some number b was selected as a *base* (or a *radix* or *scale*) for counting, and names were assigned to the numbers $1, 2, 3, \ldots, b$. Names for numbers larger than b were given by some process of combining the names for smaller numbers.

Note. Eves states that: "There is evidence that 2, 3, and 4 have served as primitive number bases" (see his page 12). He lists as an example, "the natives of Queensland." Queensland is a large state in northeastern Australia; it includes the Torres Strait Islands that stretch north towards Papua New Guinea, and it appears that this is what Eves is referencing. The Torres Strait Islanders make up part of the indigenous Australian population, along with the Aboriginal peoples of mainland Australia and Tasmania. A dominant myth is that the indigenous Australians could not count beyond four or five, and that their language for enumerations goes "one, two, three, many." Unsurprisingly, this is untrue. Details about this are given in our Supplement. Additional Numeral Systems, where a body-counting system is given for the Torres Strait Islanders. The argument for a base 2 interpretation of their counting system is based on words used for counting. The numbers 1 through

6 translate into English as "one," "two," "two one," "two two," "two two one," and "two two two." Peter Rudman in his *How Mathematics Happened: The First 50,000 Years* (Prometheus Books, 2007) proposes contracting "two two" to the single term "twotwo" so that the following base 2 interpretation results:

Base 10 Number	Aborigine Language	Base 2 Interpretation
1	one	1
2	two	2
3	two one	2+1
4	twotwo	4
5	twotwo one	4+1
6	twotwo two	4+2

Rudman himself state: "Although with this imaginative stretch it is possible to interpret this Aborigine counting as base-2, in reality it surely is two-word additive and not base-2." See Rudman's pages 56 and 57. We explore additive numeral systems in Section 1.4. Simple Grouping Systems.

Note. A *quinary scale* is a number system that has base 5. This type of counting is, no doubt, related to finger counting (a special case of body counting). A quinary number system was used in various parts of Oceania (that is, Australasia, Melanesia, Micronesia, and Polynesia). Quoting from Georges Ifrah's *The Universal History of Numbers: From Prehistory to the Invention of the Computer*, translated from French by D. Bellos, E. Harding, S. Wood, and I. Monk (John Wiley & Sons, 2000):

"You use the five fingers of the left hand to count the first five units. Then, once this number is reached, you extend the thumb of the right hand, and go on counting to 10 with the fingers of the left hand; then you extend the index finger of the right hand and count again on the left hand from 11 to 15; and so on, up to 25. The series can be extended to 30 since the fingers on the left hand are usable six times in all." (Ifrah, page 44.)
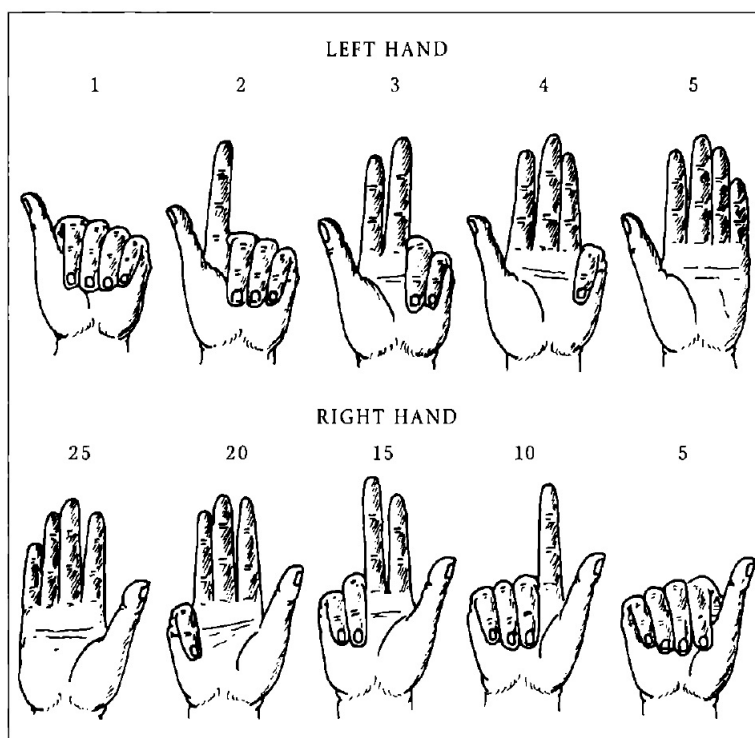
This is illustrated in Ifrah's Figure 2.11:

FIG. 2.11.

Note. A *duodecimal scale* is a number system that has base 12. It has the convenience that 12 has several divisors, so that it is easy to find certain fractional parts, such as 1/2, 1/3, 1/4, 1/6, 2/3, and 3/4. For this reason, in part, Eves comments

that duodecimal number systems "may have been used in some societies during prehistoric times, chiefly in relation to measurements." We see a remnant of this today based on the fact that we have 12 months in a year (really, though, because there are roughly 12 lunar cycles in a year) and we have 12 inches in a foot. In English, the words "dozen" (representing 12 items) and "gross" (representing 144 items) are also still with us.

Note. A *vigesimal scale* is a number system that has base 20. This type of counting is related to ancient finger and toe body-counting (this is illustrated in Figure 2.10 of Ifrah's *The Universal History of Numbers*). We'll see an example of a vigesimal number system with the Mayan number system in Section 1.7. Positional Numeral Systems. Evidence of the use of base 20 by the Celts (who lived across much of Europe from about 500 BCE to about 500 CE when they were conquered by the Romans, by which time the remaining pockets of Celts were in the British isles) is present in the French language versions of certain numbers. For example, 80 is *quatre-vingts* meaning four twenties, and 90 is *quatre-vignts-dix* meaning four twenties and ten. In English, the word "score" means twenty (as in Lincoln's "Four score and seven years ago..." of the Gettysburg address). In the pre-decimal currency of Britain (used before 1971), one pound was equal to 20 shillings.

Note. A *sexagesimal scale* is a number system that has base 60. As we'll see in Section 1.7. Positional Numeral Systems, the Babylonians used a type of sexagesimal number system; actually with only two symbols available, a symbol for 1 and

a symbol for 10, the Babylonians used a hybrid "alternating 1-for-10 and 1-for-6" replacement system. Remnants of base 60 are present in our measurement of time (60 seconds equals 1 minute, and 60 minutes equals 1 hour) and angles (60 seconds equals 1 minute, and 60 minutes equals 1 degree).

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