1.3. Finger Numbers and Written Numbers

Note. In Section 1.2. Number Bases we considered body counting as motivation for certain bases of number systems (in particular, bases 5, 10, and 20). We also mentioned "spoken numbers" (that is, the names of numbers such as the French *quatre-vingt* [four-twenties] for 80). These two ideas are united in Greenland (according to Eves on page 12) where the spoken number for 20 translates as "one man" and the spoken number for 40 translates as "two men," etc.; the implication here is that these names are based on finger and toe body counting. In this section we consider the representation of numbers using configurations of the fingers.

Note. In spite of the title of the section, we don't actually consider written numbers until the next section. The permanence of written numbers are necessary for performing computations. We mentioned the use of marks on bones to tally quantities in Section 1.1. Primitive Counting. In the next few sections we consider early written number systems, starting with those of the Babylonians and Egyptians. A written symbol representing a number is a *numeral.* As a quick philosophical comment, the objects of mathematics are IDEAS and have no existence in the physical world. A 'line' is an idea and its properties are given by the axioms of geometry; a line it not the thing we draw on paper or a white board (these are *representations* of a line). Similarly, the number five is an idea and has no physical existence. The numeral '5,' on the other hand, is a physical representation of the idea 'five.' The area of mathematics involves the study of the relationships between ideas. For example, in geometry the relationships between the ideas of points, lines, circles, planes, and so forth are studied. Arithmetic is the study of the relationships between the ideas of (whole) numbers, addition, multiplication, and equality.

Note. Eves speculates (page 13) that finger numbers (that is, representing numbers by the positioning of the fingers) "probably predates the use of either number symbols or number names." In fact the word *digit* means "finger" as well as meaning our numbers 0 through 9.

Note. In Georges Ifrah's *The Universal History of Numbers: From Prehistory to the Invention of the Computer*, translated from French by D. Bellos, E. Harding, S. Wood, and I. Monk (John Wiley & Sons, 2000), finger numbers beyond the most elementary ones are described in some detail.

"There is a much more elaborate way of counting with the hand which, from ancient times until the present day, has been used by the Latins and can also be found in the Middle East where, apparently, it may go back even further in time. It is rather like the sign language used by the deaf and [mute]. Using one or both hands at need, counting up to 9,999 is possible by this method. From two different descriptions we can reconstruct it in it entirety. ... The first was written in Latin in the seventh century by the English monk Bede ('The Venerable' [circa 672– May 26, 735]) in his *De ratione temporum*, in the chapter *de computo bel loquela digitorum* ('Counting and talking with the fingers'). The other is to be found in the sixteenth-century Persian dictionary *Farhangi Djihangiri*. There is a most striking coincidence between these two descriptions written nine centuries apart and in such widely separated places." [page 52]

"Following the fall of the Roman Empire, the same manual counting remained extraordinarily in vogue until the end of the Middle Ages . . . and played a most important part in mediaeval education. The finger counting described in Bede's *De computo vel loquela digitorum* (cited above) was extensively used in the teaching of the *Trivium* of grammar, rhetoric and logic during the undergraduate years leading to the B.A. degree, which, with the *Quadrivum* (literally 'crossroads,' the meeting of the Four Ways of arithmetic, geometry, astronomy, and music) studied in the following years leading to the M.A. degree, made up the Seven Liberal Arts of the scholarly curriculum, from the sixth to the fifteenth centuries. ... Only when written arithmetic became widespread, with the adoption of the use of Arabic numerals, did the practice of arithmetic on hands and fingers finally decline." [page 56]

On pages 53 and 54 of Ifrah's voluminous work, a figure is given explaining how units (excluding 0), tens, hundreds, and thousands are formed with the fingers (he gives both the approach by Bede and the 16th century Persian dictionary).

Note. Luca Pacioli (circa 1445–June 19, 1517) published *Summa de arithmetica, geometria, proportioni et proportionalitia* ("Summary of arithmetic, geometry, proportion, and proportionality") in 1494. The book was written in Italian (as opposed to Latin; Bede's 7th century work was in Latin). It was printed (as opposed to being copied by a scribe) and is one of the first printed mathematics books. It was intended as a textbook and a reference source. It contains the first printed description of double-entry bookkeeping. It also includes the figure given below

that describes the finger numbers based on units, tens, hundreds, and thousands. Likely this was a preferred reference for finger numbers in the Mediterranean region until the spread of the Hindu-Arabic numerals. We'll see more details on Pacioli in Section 8.5. The Fifteenth Century.
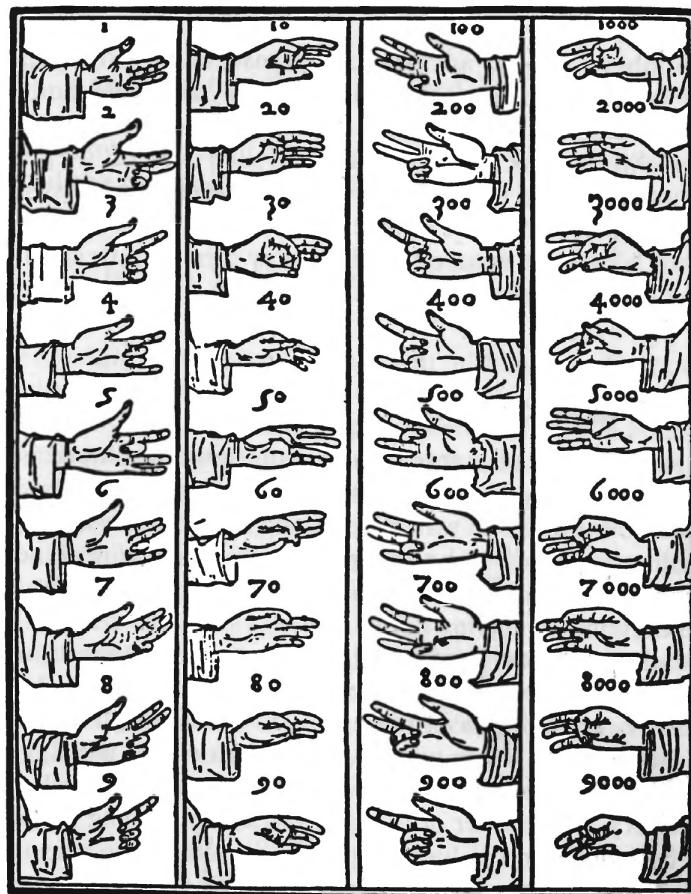
Finger numbers from Luca Pacioli's *Sūma* of 1494. The first two columns represent the left hand, the other two the right hand.

Note. Eves refers to the ongoing use of finger numbers by "primitive" peoples (a term Eves uses a bit too much; Ifrah also drops this term now and then). Eves states that some Africans, Arabs, Persians, and North and South American natives still use finger numbers (see his page 14). Well, one contemporary population that

certainly uses finger numbers is the deaf community.
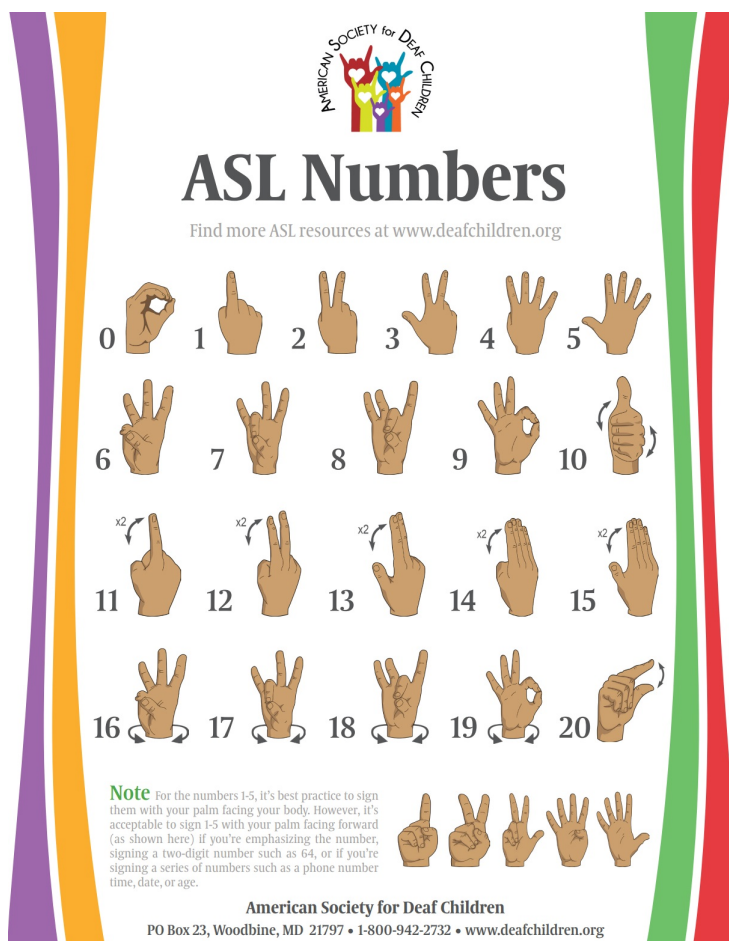
Notice that the American Sign Language ("ASL"; the above figure was accessed 5/28/2023) finger numbers for one through five differ from those given by Pacioli in the sense that Pacioli's finger numbers involve folding down fingers successively whereas the ASL numbers involve extending fingers successively. In fact, the ASL finger numbers for one through five very closely resemble the finger numbers used in parts of Oceania shown in Section 1.2. Number Bases which was used in reference to a quinary scale (that is, base 5 number system). The subtle difference is that in the Oceania numbers, the thumb goes up first (for '1') and in ASL the thumb goes up last (to complete the extension of all 5 fingers). Notice also that the larger ASL signs involve movement and, unlike the other finger numbers mentioned, are not static.

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