1.4. Simple Grouping Systems

Note. In this section we consider the Egyptian hieroglyphic numeral system (dated roughly 3000 BCE to 1000 BCE). We introduce the Babylonian numeral system as it was around 2000 BCE, which we know from preserved clay tablets. We explore earlier versions of this system in Supplement. Additional Numeral Systems, and the expression of larger numbers (larger than 59) in the Babylonian system in Section 1.7. Positional Numeral Systems. We briefly mention the Attic Greek numerals and the Roman numerals.

Definition. In a *simple grouping system* some positive integer b is selected for the base and symbols are chosen for the values 1, b, b^2, b^3, etc. Then a number is expressed using the symbols additively, each symbol being repeated the minimum number of times.

Note. The Egyptian hieroglyphic numeral system (dating as far back as 3400 BCE) is a base $b = 10$ simple grouping system. The symbols for the first seven powers of 10 are as follows:

Number	1	10	100	1,000
Hieroglyph	𓏺	𓎆	𓍢	𓆼
Object	vertical staff	heel bone	coil of rope	lotus flower

Number	10,000	100,000	1,000,000
Hieroglyph	𓂭	𓆐	𓁨
Object	pointing finger	tadpole	man in astonishment

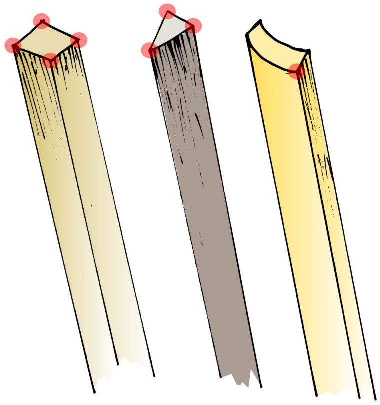
The fonts used here are based on the LaTeX package hieroglf. Eves uses slightly different font (see his page 15).

Note. As an example, we write 13,015 in the Egyptians hieroglyphs:

$$13,015 = 1(10^4) + 3(10^3) + 0(10^2) + 1(10) + 5 =$$ 𓂭𓆼𓆼𓆼𓎆𓏾

Note. Egyptian hieroglyphics were mostly used for inscriptions on stone monuments. For writing on other surfaces (papyrus, wood, and pottery), a cursive script called *hieratic* was developed. This is discussed in Supplement. Additional Numeral Systems.

Note. Information on Babylonian numerals are preserved because they recorded work on clay tablets, may of which have survived. Such Babylonian writing is known as *cuneiform*. It involves pressing a stylus into the soft clay. Different characters are made by tilting the stylus at different angles. There are two basic numerals, representing 1 and 10. These are formed by using a stylus that has an end the shape of a sharp isosceles triangle. The 1 character is made by tilting the stylus somewhat from the perpendicular and pressing the sharp angle of the triangle into the soft clay. The 10 character is similarly made, but by pressing a base angle of the triangle into the clay.

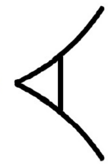


(Left) From the cuneiform.neocities.org website on writing cuneiform. These are types of writing styli used in cuneiform writing. The center stylus is used for making numerals (the red circles mark the appropriate corners). (Right) From the Cuneiform Digital Library Initiative, which has detailed information on creating cuneiform writing. (Both accessed 5/20/2023)

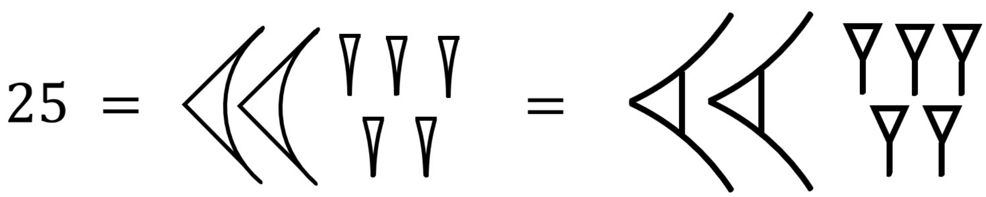
The soft clay tablets were baked to harden them, thus preserving them. Between 2000 BCE and 200 BCE numbers between 1 and 59 were expressed in the additive simple grouping system. We'll explore numbers 60 and larger in Section 1.7. Positional Numeral Systems.

Note. The two symbols as given in Eves and in Peter Rudman's *How Mathematics Happened: The First 50,000 Years* (Prometheus Books, 2007) are:

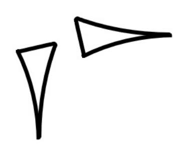
1 (Eves)  10 (Eves) 

1 (Rudman)  10 (Rudman) 

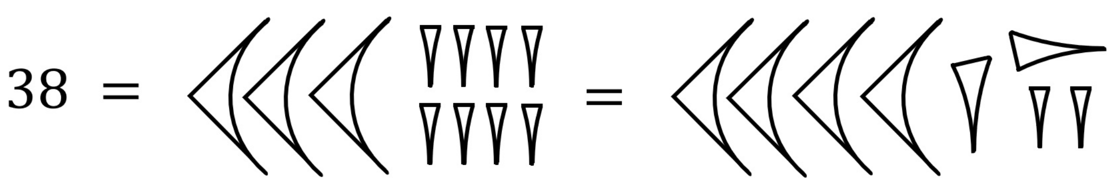
In repeating the 1's, smaller versions of the character may be used. It also seems to be common practice to overlap the characters when there are several of them repeated. As a consequence, we represent $25 = 2(10) + 5$ as:



Note. An additional illustration of the Babylonian cuneiform numerals is the clay tablet "Yale Babylonian Collection item number 7289." It contains as estimate of $\sqrt{2}$ which is accurate to 6 decimal places. Details are given in my online notes for the historical component of Introduction to Modern Geometry (MATH 4157/5157) on Section 1.6. A Remarkable Babylonian Document.

Note. The Babylonians also used a subtraction symbol: 

This allowed them to use less characters than in a simple additive system (similar to Roman numerals). For example, $38 = 3(10) + 8 = 4(10) - 2$ can be written as:



Note. The Attic (or "Herodianic") Greek numerals were developed some time before 300 BCE. They make up a simple grouping system with base 10, with a special symbol for 5. The symbols are:

Number	1	5	10	50	100	500	1000	5000	10000	50000
Name	-	pente	deka	-	hekaton	-	kilo	-	myriad	-
Symbol	Ι	𐅃	Δ	𐅄	Η	𐅅	Χ	𐅆	Μ	𐅇

Notice that the symbol for 5 (a symbol based on Π, the first letter of "pente") is modified and used in combination with the base 10 symbols to produce intermediate characters). The simple grouping method (modified from base $b = 10$ to include the special symbols based on "pente") implies the following example:

$$2857 = \underbrace{\text{ΧΧ}}_{2000}\underbrace{\text{𐅅ΗΗΗ}}_{800}\underbrace{\text{𐅄}}_{50}\underbrace{\text{𐅃ΙΙ}}_{7}$$

Note. Finally, we consider the Roman numerals, which are familiar to many of us (in particular, for their appearance as the copyright year displayed at the end of a movie). The basic symbols, along with special symbols related to multiples of 5 are:

Number	1	5	10	50	100	500	1000
Symbol	I	V	X	L	C	D	M

The symbols are arranged left to right from larger value to smaller value, with one exception. If a single numeral with a smaller value is placed immediately before a numeral for a larger value, then the additive system is modified and the smaller value is subtracted from the larger value (similar to the Babylonian system, though there is no particular character for subtraction; it is based on the position of the symbols). This subtractive principle comes with one additional rule: I can only precede V or X, X can only precede L or C, and C can only precede D or M. According to Eves, the subtractive principle "was use only sparingly in ancient and medieval times" and the "fuller use of this principle was introduced in modern times" (page 16). So we could represent 9 as VIIII (in premodern notation) or IX (in modern notation). As with the Babylonian use of subtraction, the modern version requires fewer symbols. As another example we have (in premodern and modern notation):

$$1944 = \text{MDCCCCXXXXIIII} = \text{MCMXLIV}.$$

In Latin "hundred" is *centum* and "thousand" is mille, explaining the use of the symbols C and M. Eves gives some other (apparently speculative) ideas about the origins of the other symbols (see Eves' page 17).

Revised: 5/21/2023