1.5. Multiplicative Grouping Systems

Note. In this section we define a multiplicative grouping system and illustrate it with the standard Chinese numerals.

Definition. In a *multiplicative grouping system* some integer $b > 1$ is selected for the base, a first set of symbols are chosen for the values $1, 2, \ldots, b - 1$, and a second set of symbols are chosen for $b, b^2, b^3, \ldots$. Then a number is expressed using the symbols multiplicatively, with a symbol from the first set giving the number of units from the second set (followed by an understood summation).

Note. With $b = 10$, the first set of symbols as $\{1, 2, \ldots, 9\}$, and the powers of $b = 10$ (in order) as $\{a, b, c, \ldots\}$, we would represent 5625 as $5c6b2a5$. We interpret this as $5626 = 5 \times 10^3 + 6 \times 10^2 + 2 \times 10 + 5$. Since there is no symbol for zero, in this example we would represent 5007 as $5c7$. Our own Hindu-Arabic numeral system is a modified version of a multiplicative grouping system. By employing the addition symbol 0, we can use the symbols from the second set to introduce a positional notation from a multiplicative grouping system; see Section 1.7. Positional Numeral Systems for more details. For example, $3d4c1a7$ in the multiplicative grouping system translates into 34,017.

Note. Chinese numerals give an example of a multiplicative grouping system. The following historical information is from George G. Joseph's *Crest of the Peacock: Non-European Roots of Mathematics*, Third Edition (Princeton University Press,

2011); see pages 198–200. There are four main types of Chinese numerals: (1) the standard (or modern) numerals, which we concentrate on here, (2) the official numerals which are decorative versions of the standard numerals and are used on legal documents and banknotes, (3) the commercial numerals can be written quickly are today are found on price tags or bills, and (4) the stick (or rod) numerals. The standard numerals have their origin in number-words and date from about the third century BCE. The commercial numerals date from about the sixteenth century. The stick numerals have been around since at least the second century BCE. The earliest known Chinese numerals are on "oracle bones" from the Shang dynasty and date from between 1500 BCE and 1200 BCE. The bones were used by Shang nobles in rituals appealing to their ancestors for advice on practical matters (such as best times for travel and planting). The bones recorded numbers from 1 to 30,000. The numbers 1 to 10 were written as (from page 199 of *Crest*):

1	2	3	4	5	6	7	8	9	10

The standard numerals are a descendant of this ancient Shang system, and similarities are obvious (for 1, 2, 3, 7, and maybe 8)

Note. The Chinese numerals as given by Eves are:

Value	1	2	3	4	5	6	7	8	9
Symbol	一	二	三	四	五	六	七	八	九
Mandarin	yī	èr	sān	sì	wǔ	liù	qī	bā	jiǔ
Cantonese	jat	ji	saam	sei	ng	luk	cat	baat	gau

Value	10	100	1000
Symbol	十	百	千
Mandarin	shí	bǎi	qiān
Cantonese	sap	baak	cin

Since these are an example of a multiplicative grouping system, then the number 5625 is written as (reading from top to bottom):

五
千 } 5000

六
百 } 600

二
十 } 20

五 } 5

Note. The Japanese numerals are the same as the Chinese numerals (and the same as the Sino-Vietnamese, though they differ from the native Vietnamese), though then names are based on Japanese number names. The early Chinese and Japanese writings were recorded on bamboo strips. The strips were laid side by side and tied together. Since the strips were narrow, the writing on them was done vertically. This custom continued into modern times, when silk and paper became available as a writing surface.

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