1.6. Ciphered Numeral Systems

Note. In this section we define a "ciphered numeral system" and illustrate it with the system example used by the Greeks as far back as 450 BCE.

Definition. A *ciphered numeral system* has a base $b > 1$ and a set of symbols for

$$1, 2, \ldots, b-1; b, 2b, \ldots (b-1); b^2, 2b^2, \ldots, (b-1)b^2; \ldots.$$

Note. A ciphered numeral system requires many symbols (infinitely many, technically; in particular, one for every multiple and power of the base b). However, it does allow for a relatively small representation of numbers. As an example, the alphabetic Greek numeral system (or "Ionic Greek numeral System") uses 27 characters and gives easy representations of numbers up to 999.

Note. The 27 symbols in the alphabetic Greek numeral system are as follow.

1	α	alpha	10	ι	iota	100	ρ	rho
2	β	beta	20	κ	kappa	200	σ	sigma
3	γ	gamma	30	λ	lambda	300	τ	tau
4	δ	delta	40	μ	mu	400	υ	upsilon
5	ε	epsilon	50	ν	nu	500	ϕ	phi
6	$\digamma$	digamma	60	ξ	xi	600	χ	chi
7	ζ	zeta	70	o	omicron	700	ψ	psi
8	η	eta	80	π	pi	800	ω	omega
9	θ	theta	90	$\Qoppa$	koppa	900	$\Sampi$	sampi

These are the familiar 24 letters of the Greek alphabet, along with three "obsolete" symbols, the *digamma*, *koppa*, and *sampi*. These are, respectively, ϝ, ϟ, and ϡ. The fonts used here are based on the LaTeX package teubner. Eves uses a different font for digamma, which resembles a type of "S" (see his page 19).

Note. Some examples of representations with the alphabetic Greek numeral system are:

$$12 = \iota\beta, \;\; 21 = \kappa\alpha, \;\; 247 = \sigma\mu\zeta, \;\; 556 = \phi\nu\digamma, \;\; 999 = \sampi\koppa\theta.$$

No big deal, right? Then consider solving this *without* converting to our Hindu-Arabic numerals: $\rho\pi\alpha \times \psi\koppa\delta$.

Note. As explained in Problem 1.3(b), primes were often used to represent thousands, so that $1000 = \alpha'$, $2000 = \beta'$,..., $9000 = \theta'$. The number 10,000 (or "*myriad*") was denoted "M," then the multiplication principle was used for multiples of 10,000: $20{,}000 = \beta M$, $300{,}000 = \lambda M$, $4{,}000{,}000 = \upsilon M$, and $100{,}000{,}000 = MM$.

Note. Other examples of ciphered numeral systems are the Egyptian hieratic, Coptic, Hindu Brahmi, Hebrew, Syrian, and early Arabic. The last three are alphabetic, like the alphabetic Greek numeral system. These are explored in Supplement. Additional Numeral Systems.

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