1.8. Early Computing

Note. In this section we discuss the medium on which math was recorded, other than clay tables and stone (namely, papyrus and parchment) and explain that this stuff was too valuable to be used for scratch work, thus delaying the historical development of some of the techniques we use for elementary calculation today. We also consider the first types of abacuses and illustrate their use for Roman numerals.

Note. The techniques by which we perform multiplication and long division (by hand) "were developed as late as the fifteenth century" (Eves, page 21). Eves mentions two common reasons given for this late development: (1) "mental difficulties" and (2) physical difficulties. He discounts the first of these, since the older number systems could be used much as we do multiplication and division with the Hindu-Arabic numerals. For example, in a ciphered numeral system our modern primary-school techniques can be applied provided addition and multiplication tables have been memorized (as illustrated by multiplication examples in Supplement. Additional Numeral Systems). On the other hand, the physical difficulties were more substantial. We take for granted the wide availability of writing utensils and writing surfaces (namely, cheap pencils and paper). Without this, the development and learning of involved arithmetic processes would be slowed. Paper made of rag (which is available still today) was handmade, expensive, scarce, and did not appear in Europe until the twelfth century.

Note. *Papyrus* is similar to thick paper, but is made from the pith of the papyrus plant, which was abundant along the Nile River delta (and of course this is the source of our word "paper"). The stems of the papyrus reed were cut into strips that were laid out side by side, with a second layer on top of the first at a right angle. The result was soaked in water, then pressed out and dried. Finally, the sheets had to be smoothed by rubbing them. As a result, papyrus was valuable and was not used for scratch-work math. It was introduced to the Greeks by 650 BCE and was the medium on which they recorded much of their work. In fact, you can still by it in 6 by 8 inch sheets from Amazon.com for less than a dollar per sheet (it is mean for craft projects; this bit of information is from summer 2023)! We will explore the content of the Rhind Papyrus in Section 2.10. Egypt: A Curious Problem in the Rhind Papyrus. It dates from 1550 BCE and is from Egypt (though it is housed in the British Museum, London). It contains information on arithmetic, algebra, and geometry.

Image of the Rhind Papyrus from the Wikipedia Rhind Mathematical Papyrus webpage (accessed 6/7/2023)

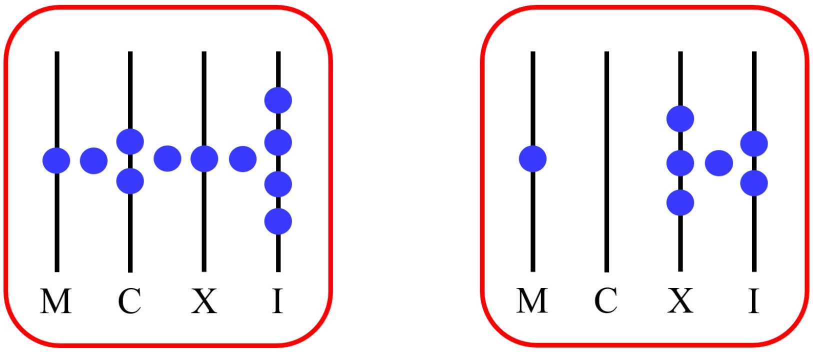
Note. Even more expensive then papyrus, were animal skins used as a writing medium. *Parchment* is made from animal skin (usually sheep, lambs, or goats) that has been scraped clean and dried under tension. *Vellum* is the skin of a young animal (often a calf) prepared in a similar way. These have been used as a writing surface for over two millennia (the Greek historian Herodotus mentions writing on skins as common in his time, the 5th century BCE, though "skin" may include leather, which is prepared differently from parchment and vellum). Paper came into use by the 15th century, and parchment was replaced by the cheaper option. Most of this history is from the Wikipedia page on Parchment (accessed 6/7/2023). Because of the expense, it was common practice to wash the ink off of old parchment manuscripts, scrape them clean, and reuse them (sometimes rotating the pages 90 degrees to avoid writing directly on top of the old writing). A manuscript produced in this way is a *palimpsest*. We will see a famous example of a palimpsest in "Supplement. Archimedes: 2,000 Year Ahead of His Time," online in PowerPoint with an online transcript in PDF.

Note. The Romans used a writing stylus to make etchings on a small board covered with a coat of wax. Surprisingly, you can find these for sale on Amazon.com as well, where the next image is from (accessed 6/7/2023).

Note. An *abax* (or "sand tray") was a table covered with sand on which students could practice writing, geometry and calculations; these were common in Greece. It is the predecessor of the abacus. Objects, such as pebbles, were added for counting and moved around between columns to perform place-value arithmetic. Notice that an "abacus" is a table on which counters are moved around, not a system of wires along which beads slide (this is a "Chinese abacus"). This note is based on the Wikipedia page on Sand Tables; this is also the source of the following image.

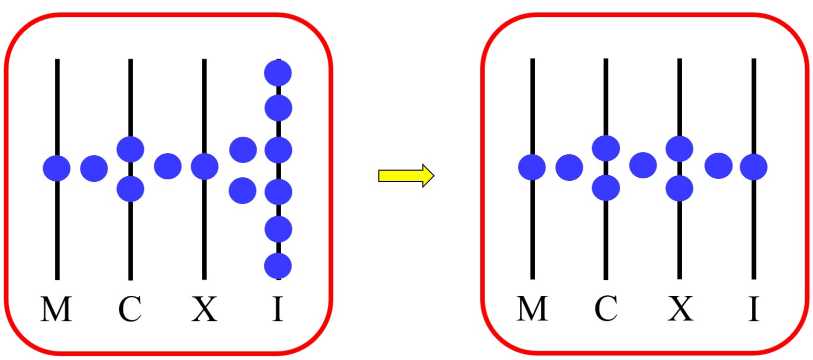
This image is a drawing based on a scene painted on the Darius Vase, which was made aound 330 BCE in Greece. It shows a tax collector calculating on his abacus.

Note. We now illustrate the use of an elementary form of abacus as applied to Roman numbers. The abacus (i.e., table top) has four vertical parallel lines labeled left to right as M, C, X, and I. Then counters are placed on the lines such that the indicated numbers sum to the desired entry. To reduce the number of counters (and to mimic the Roman numerals), whenever a vertical line has five counters on it, they are replaced with with a single counter placed to the left of that line. When two counters like this are between two vertical line, they are replaced with a single counter on the line to the left (i.e., they “‘carry over”). Consider the sum of MDCCLXIX and MXXXVII. We start by “entering” the first number on the abacus as follows (we also show how the second number would be entered on the abacus, though this is not needed for the computation).



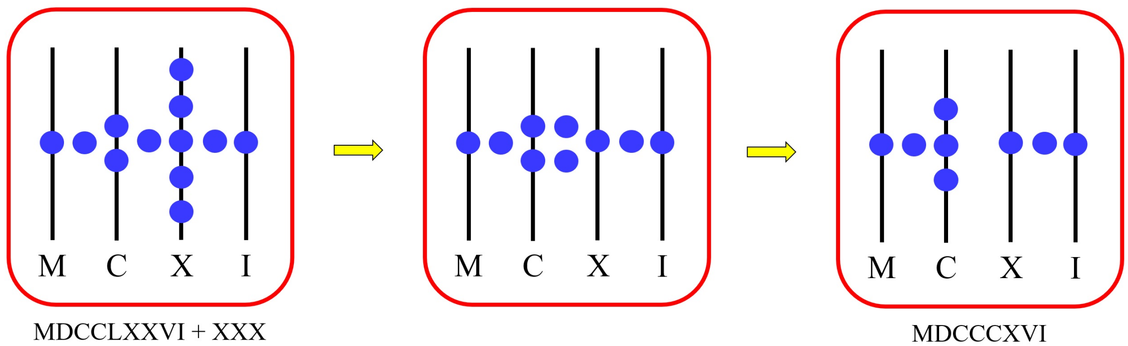
MDCCLXIX MXXXVII

To add, we first consider the VII part of MXXXVII. This gives two more counters on the I line and one more between the X and I lines. But then there are six counters on the I line, so we remove five of them and place another counter between the X and I lines. Now, there are three counters between the X and I lines, so we “carry over” two of them as a single counter onto the X line.

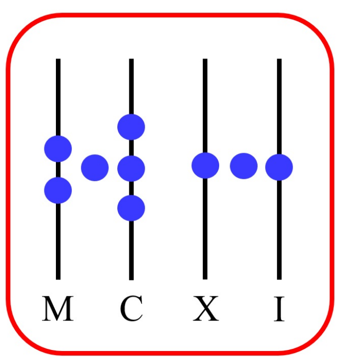


MDCCLXIX + VII MDCCLXXVI

Next, we add the XXX part of MXXXVII. This means putting three more counters on the X line, for a total of five. These are replaced with a single counter between the C and X lines giving a total of two counters between these lines. So we carry over the two of them as a single counter on the C line.



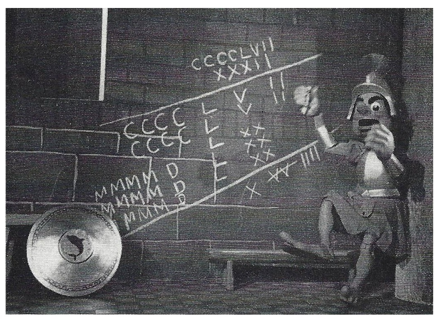
Finally, we add the M part of MXXXVII. We place an additional counter on the M line. The configuration of the counters satisfy our rules without any additional adjustment, so we now have that MDCCLXIX + MXXXVII = MMDCCCVI. It's not clear, but it doesn't look like the puppet get the correct answer.

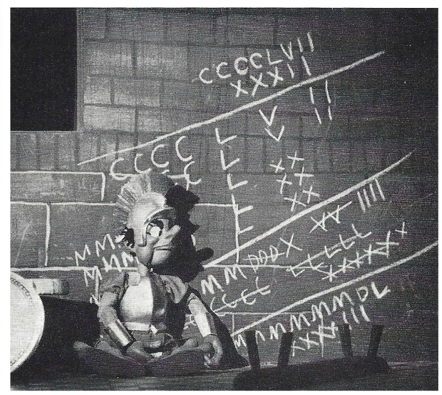


MDCCLXIX + MXXXVII = MMDCCCXVI

When subtracting, the processes of replacing 5 counters on a line for one counter between lines, and replacing two counters between a line with one counter on the left line (that is, carrying over) are replaced with the inverse process of "borrowing."

Note. We do not address multiplication with Roman numerals here. It can be exceeding involved! The conversions of two V's into an X (and the analogous substitutions), along with the representations of 4, 40, 400, etc., combine to make the use of a small multiplication table and the carrying over process quite difficult when multiplying. This is illustrated in the following images of a Roman soldier marionette attempting to compute 457×32. The solution is 14,624 or, in Roman numerals, $\bar{\text{X}}$M$\bar{\text{V}}$DCXXIV ($\bar{\text{V}}$ represents 5,000 and $\bar{\text{X}}$ represents 10,000).





Using Roman Numerals seems difficult to those of us who work arithmetic problems with Arabic numerals. To illustrate, a *marionette* (puppet) Roman soldier confidently starts solving a multiplication problem, *top*. He becomes angry as he struggles with it, *above*, and then gets the wrong answer, *below*. Actually, Romans and others who worked with Roman numerals usually used an abacus to multiply.

Ormond Gigli, *Life* © 1957 Time, Inc.

These pictures are scanned from the article on "Roman Numerals" (which was written by our textbook author, Howard Eves) in *The World Book Encyclopedia*, Volume 16 Q–R, page 395 (Chicago: Field Enterprises Educational Corporation, 1969), but apparently they first appeared in *Life Magazine* in 1957.

Revised: 6/8/2023