2.10. Egypt: A Curious Problem in the Rhind Papyrus

Note. In this section, we state Problem 79 of the Rhind Mathematical Papyrus. It involves summing a geometric progression with ratio $r = 7$. We give various interpretations throughout history of the "word problem" associated with the given numbers.

Note. As seen in Section 2.7. Egypt: Sources and Dates (see Note 2.7.D), the Rhind Mathematical Papyrus dates to 1550 BCE, contains 87 problems (as given by modern enumeration), and is broken into three parts, where Part III is "Miscellany" and contains Problems 61 to 84 (and "Numbers" 85, 86, and 87). The "curious problem" is Problem 79 in Part III. In the book Arnold B. Chace (with assistance of Henry P Manning), *The Rhind Mathematical Papyrus*, British Museum 10057 and 10058, Volume I, Free Translation and Commentary (Mathematical Association of America, 1927), the following translation of Problem 79 is given on page 112 (*The Rhind Mathematical Papyrus* is online on the Wikipedia.org (accessed 8/13/2023).

> *Problem 79*
>
> *Sum the geometrical progression of five terms, of which the first term is 7 and the multiplier 7.*
>
> *The sum according to the rule.* Multiply 2801 by 7.

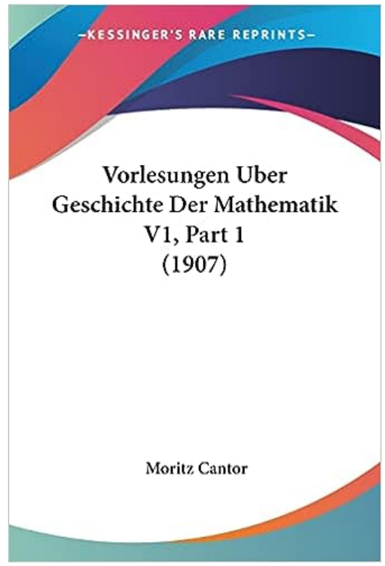
1	2801
2	5602
4.	11204
Total	19607.

The sum by addition.

houses	7
cats	49
mice	343
spelt	2401
hekat	16807
Total	19607.

Notice that the numbers in the second table are consecutive powers of 7. On page 30 of *The Rhind Mathematical Papyrus*, Chace explains that the first computation can be interpreted as the sum of n terms of a geometric progression with ratio r and first term $a = r$, where $r = 7$ and $n = 5$. The relevant formula is then $r + r^2 + r^3 + \cdots + r^n = r(1 + r + r^2 + \cdots + r^{n-1})$. So with $r = 7$ and $n = 5$, we have $7 + 7^2 + 7^3 + 7^4 + 7^5 = 7(1 + 7 + 7^2 + 7^3 + 7^4) = 7(1 + 2800) = 7(2801)$. This is the reason that the first computation considers the product of 2801 and 7 (though the computation of the value $7 + 7^2 + 7^3 + 7^4 = 2800$ is not shown; though it can be surmised from the numbers given in the second computation). Notice that in the first computation, 7 has been written as a sum of powers of 2: $7 = 1 + 2 + 4$. In this way, the Egyptian technique of multiplication given in Section 2.8. Egypt: Arithmetic and Algebra is used (see Note 2.8.A).

Note. Moritz Cantor (August 23, 1829–April 9, 1920) was a German historian of mathematics who published Vorlesungen über Geschichte der Mathematik ("Lectures on the History of Mathematics") in four volumes between 1880 and 1908. It was the most comprehensive work on the history of math at the time. In 1907, Part 1 of Volume 1 was reprinted, and this reprint can still be bought today.



Images from the MacTutor biography page for Moritz Cantor (left) and Amazon.com (right); accessed 8/13/2023.

As stated by Eves on his page 56, Cantor's interpretation of Problem 79 (in 1907) was: "An estate consisted of seven houses; each house had seven cats; each cat at seven mice; each mouse ate seven heads of wheat ["spelt"]; and each head of wheat was capable of yielding seven hekat measures of grain. Houses, cats, mice, heads of wheat, and hekat measures of grain, how many of these were in the estate?" Apparently this is Volume 1 of Cantor's work (so it appeared in 1880), but Eves gives the 1907 date of the reprint.

Note. In 1202 CE (some 2700 years after the Rhind Papyrus), Leonardo of Pisa ("Fibonacci") stated a related problem in his *Liber abbaci*. We give extensive details on Leonardo and the *Liber abbaci* in Section 8.3. Fibonacci and the Thirteenth Century and Supplement. Leonardo of Pisa (Fibonacci) and the *Liber abbaci*. As stated on page 438 of *Fibonacci's Liber Abaci: A Translation into Modern English of Leonardo Pisan's Book of Calculation* (Springer, 2002) by Laurence Sigler (the first complete English translation of *Liber abbaci*), the problem is called "Seven Old Men Go to Rome" and is:

> Seven old men go to Rome; each of them has 7 mules, and on each mule there are 7 sacks, and in each sack there are 7 loaves of bread, and for each 49 loaf of bread there are 7 knives, and each knife has seven scabbards. The sum of all the aforesaid is sought.

This is effectively Problem 79 of the Rhind Papyrus, but it includes one additional power of 7 (so that the sum is $19607 + 7^6 = 19607 + 117649 = 137256$; 19607 is the solution to Problem 79).

Note. Some 600 years after *Liber abbaci*, the following English nursery rhyme, "As I was going to St. Ives," appears.

> As I was going to St. Ives,
> Upon the road I met seven wives;
> Every wife had seven sacks,
> Every sack had seven cats,
> Every cat had seven kits:
> Kits, cats, sacks, and wives,
> How many were going to St. Ives?

Again, this is effectively Problem 79 of the Rhind Papyrus, but it includes one *less* power of 7 (so that the sum is $19607 - 7^5 = 19607 - 16807 = 2800$). But the question is slightly different, and arguably could include the "I" who is also going to St. Ives so that the total is 2801 (variants of the nursery rhyme modify the second line to read "I met a man with seven wives," so that the total going to St. Ives is then 2802). Alternatively, one could argue that on the "I" is *going to* St. Ives and the others are *coming from* St. Ives, so that the answer is 1. Nonetheless, according to the Wikipedia webpage for "As I was going to St. Ives" (accessed 8/13/2023), the nursery rhyme dates from at least 1730. If "As I was going to St. Ives" is still circulating out there, then this is a variant of a mathematical puzzle that has been around at least 3500 years! As some circumstantial evidence that it is still circulating and is somewhat well-known, Amazon.com offers a notebook with an image by Arthur Rackham on the cover, which illustrates the story of the nursery rhyme.

The picture shows the narrator, "I," the man (with the smaller hat), the seven wives, many cats, and very many kits! You may have heard of English illustrator Arthur Rackham (September 19, 1867–September 6, 1939) as an illustrator of

Charles Dickens' *A Christmas Carol* (a 1915 edition published by William Heinemann, London) and Clement Clark Moore's *The Night Before Christmas* (a 1931 edition published by Geoege G. Harrap, London), though he illustrated many more works.

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