2.3. Babylonia: Commercial and Agrarian Mathematics

Note. In this brief section, we discuss clay tablets concerning inventories of agricultural material, the existence of mathematical tables on the clay tablets, and there applications to division and compount interest.

Note. We saw in Section 1.7. Positional Numeral Systems that the Babylonians used a base 60 positional system. The clay tablets reveal a high level of computational ability within this system. The tablets include arithmetic calculations based on transactions related to farm deliveries. As mentioned in Supplement. Additional Numeral Systems, even the Sumerians (who preceeded the Babylonians) made clay records like this where they would record the number of an object traded and include a pictograph of the object itself. In fact, "[t]he tablest show that the ancient Sumerians were familiar with all kinds of legal and domestic contracts, like bills, receipts, promissory notes, accounts, both simple and compound interest, mortgages, deeds of sale, and guaranties" (Eves, page 41).

Note. Of the 400-odd mathematical clay tablets, over half of them contain mathematical tables. There are multiplication tables, tables of reciprocals, tables of squares and cubes, and tables of exponentials. The tables of reciprocals were used to convert division to multiplication (remember that, theoretically at least, there really is no such thing as division but instead there is multiplication by multiplica-

tive inverses). The tables of exponentials were likely used for compound interest computations (as mentioned above).

Note. When considering reciprocals, it is desirable that it have a finite expansion when expressed as a radix fraction (that is, as a "decimal expansion" base b; see Problem Study 1.6). A number n which has a reciprocal $1/n$ that has a base 60 finite sexagesimal expansion is sexagesimally *regular*. For example, $1/3$ is sexagesimally regular since $1/3 = (0.t)_{60}$ where $t = 20$ (so the sexagesimal expansion consists of only one digit). Of ocurse, $1/3$ is not decimally regular since $1/3 = 0.3333\cdots = 0.\overline{3}$. The vast majority of Babylonian clay tablets of tables of reciprocals contain only reciprocals of sexagesimally regular numbers. It is to be shown in Problem Study 2.1 that n is sexigesimally regular if and only if $n = 2^a 2^b 5^c$ for some nonnegative integers a, b, c. Exponential tables and their applications to compound interest is addressed in Study Problem 2.2.

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