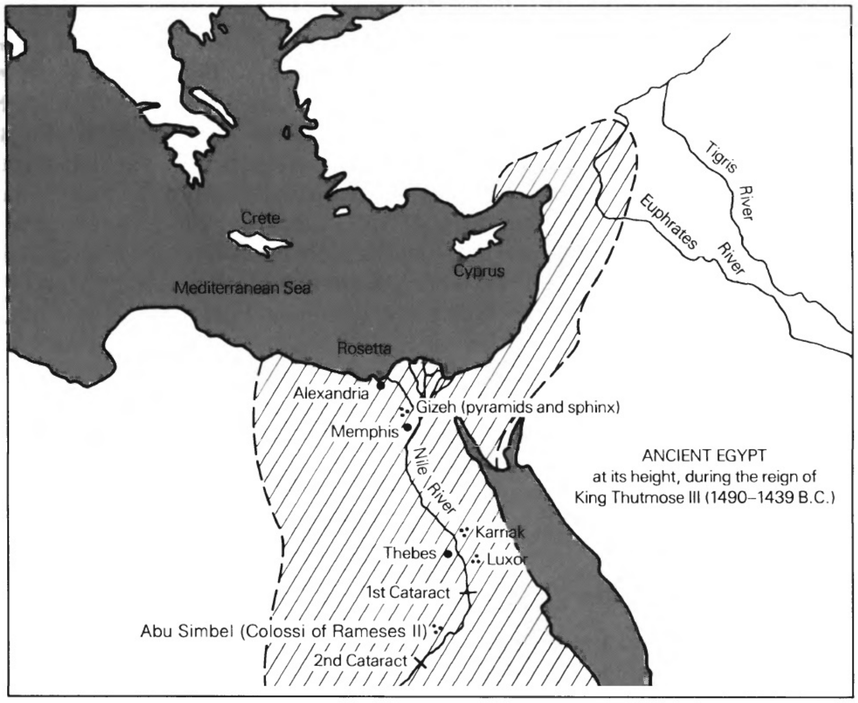
2.7. Egypt: Sources and Dates

Note. This section is a paraphrasing of Eves' presentation. We compare and contrast Babylon and Egypt, and give a list Egyptian sources of a mathematical nature.

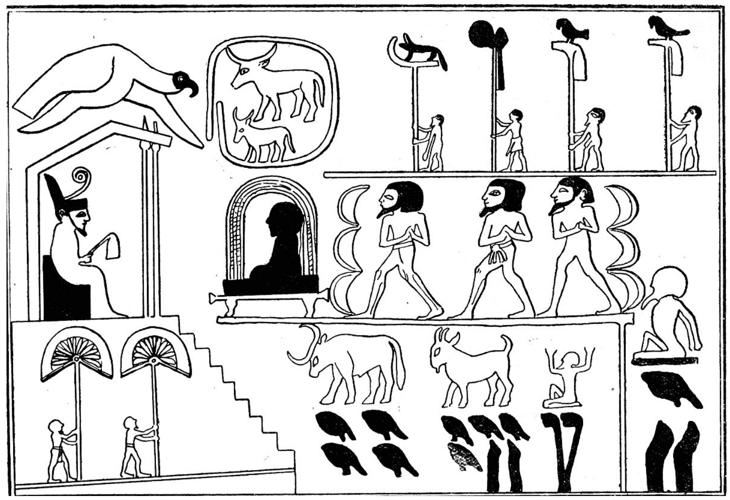
Note. Babylon was located on a number of caravan trading routes. This lead to economic development (and the need for more mathematical knowledge), but also exposed them to turmoil, which lead to a succession of empires. On the other hand, Egypt was in semi-isolation and naturally protected from foreign invasion, leading to a relatively peaceful sequence of Egyptian dynasties. In fact, ancient Egyptian mathematics never reached the level of that of the Old Babylonians. Both Babylon and Egypt were theocracies that were ruled by bureaucratic leaders and temple priests. Both societies had a large slave class. In Babylon, the slave population was made up of those from a defeated empire who were enslaved by a conquering empire. In Egypt, slaves were taken from foreign lands after military conquests. According to Eves (page 47), slave labor played a role in maintaining irrigation systems and engineering control of rivers in both societies (the erratic Tigris and Euphrates Rivers in Babylon, and the Nile in Egypt). Eves also claims that slave labor built the ziggurats (rectangular, stepped towers of religious importance) in Babylon and the great pyramids in Egypt. However, archaeologists today disagree with the claim that the Egyptian pyramids were built with slave labor. Your instructor speculates that Eves' claim about the Babylonian ziggurats is also questionable.



A portion of Eves' map of ancient Egypt, page 48.

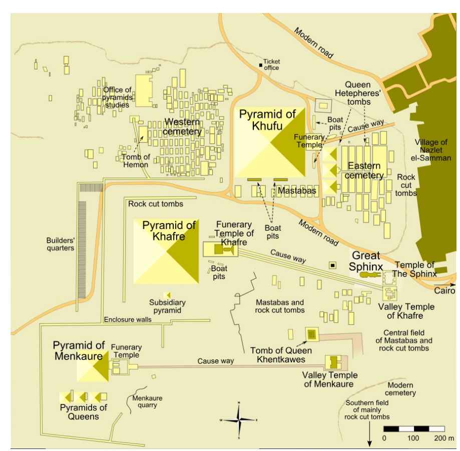
Note. Ancient Egyptian's veneration for their dead led to mummification of the dead and construction of very durable tombs and temples with inscribed walls. This, combined with the later development of written records on papyrus and the dry climate of the region, has given us a large amount of historical information on their culture. We now give a chronological list of some of these sources which have some mathematical content.

Note 2.7.A. The Narmer macehead is a decorative stone mace head dating from around 3100 BCE. It is housed in the Ashmolean Museum of Art and Archaeology in Oxford, England. It is of mathematical interest because it includes a hieroglyphic description of the plunder resulting from a military victory. In the image below right (bottom center of the image), there is recorded that 400,000 cattle, 1,422,000 goats, and 120,000 human captives were taken.



Images from AncientEgypt.org (left) and Wikipedia webpage on the Narmer Macehead (right); both websites accessed 8/10/2023.

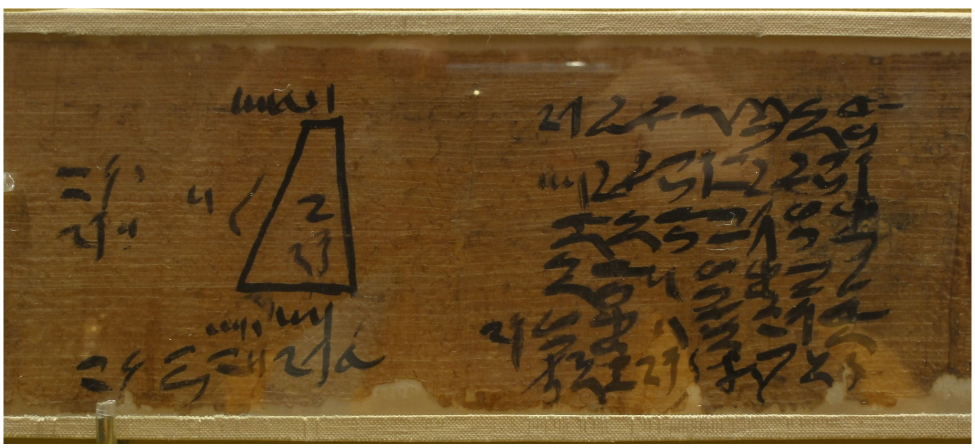
Note 2.7.B. The Great Pyramid of Giza is the largest of the Egyptian pyramids. It was built around 2600 BCE as a tomb for pharaoh Khufu (or "Cheops").



The Giza pyramid complex from the American Physical Society webpage (left) and a map of the region from Wikipedia webpage on the Great Pyramid of Giza (right). Websites accessed 8/10/2023.

The Giza pyramid complex is dominated by the Great Pyramid, the Pyramid of Khafre (the tomb of pharoah Khafre, or "Chephren," who ruled from about 2558 BCE to 2532 BCE), and the Pyramid of Menkure (or "Mycerinus"; the smallest of the three main pyramids, probably constructed in the 26th century BCE). Khafre and Menkure were the two pharoahs who immediately succeeded Khufu. See the Wikipedia webpages on Pyramid of Khafre and Pyramid of Menkaure for more details (accessed 8/10/2023). Construction of the pyramids certainly required some knowledge of mathematics, surveying, and engineering (but *did not* require space aliens!). The Great Pyramid covers thirteen acres and contains over two million blocks, which average 2.5 tons in weight. Most of the blocks are from sandstone from the other side of the Nile River. Some of the chamber roofs are made of fifty-four-ton granite blocks which were hauled from 600 miles away, and were sitting 200 feet above the ground from where they were quarried. Eves reports the relative errors of the sides of the (square) base and the relative error of the (right angled) corners are one in 14,000 and one in 27,000, respectively. It took 30 years to construct. It is 481 feet tall, the square base is 756 feet per side, and the slope of the sides are 52°. By contrast, the Pyramid of Khafre is 448 feet tall, has a base measuring 448 feet per side, and the slope of the sides is 53°. The Pyramid of Menkure is 215 feet tall, has a 200 foot square base, and slope of sides of 51°. The sources for this note are the mentioned Wikipedia pages and Eves' pages 49 and 50.

Note 2.7.C. The Moscow Mathematical Papyrus dates from roughly 1850 BCE. It was purchased by Vladimir Golenishchev in 1892 or 1893 in Thebes, Egypt (it is sometimes called the "Golenishchev Mathematical Papyrus"). It is now housed in the Pushkin State Museum of Fine Arts in Moscow (hence the more common name). It is 18 feet long and between 1.5 and 3 inches wide. In 1930, V. V. Struve divided it into 25 problems with solutions (though the first two problems are damaged and unreadable). A table describing the problems is given on the Wikipedia webpage on the Moscow Mathematical Papyrus (accessed 8/10/2023), the main source for this note. The Moscow Mathematical Papyrus contains "pefsu problems" (which measure the strength of beer), "Baku problems" (related to the output of workers), and geometry problems (concerning areas of triangles, surface areas of a hemisphere, and the volume of a frustum). Problem Studies 2.14 and 2.15 give some of these problems.



This photo of part of the Moscow Papyrus by Charles Dorce is from the The Mathematical Tourist webpage on the Moscow Mathematical Papyrus (accessed 8/10/2023).

Note 2.7.D. The Rhind Mathematical Papyrus is the best known example of ancient Egyptian mathematics. It was purchased by Alexander Rhind in 1858 in Luxor, Egypt. It is housed in The British Museum, which dates it at 1550 BCE (though Eves gives a date of 1650 BCE). In fact, you can see the Rhind Papyrus on The British Museum website (accessed 8/11/2023). It is 13 inches tall and over 16 feet long (though it is in pieces and some small fragments are held in the Brooklyn Museum in New York). So it is larger and less old than the Moscow Papyrus. The Rhind Papyrus is written in the hieratic script (see Supplement. Additional Numeral Systems for more on hieratic numerals) by the scribe Ahmes who copied an earlier work (as stated in the opening paragraphs of the papyrus; it is sometimes called the "Ahmes Papyrus"). Translation started in the late 1800s. The translation was published in Thomas Peet, *The Rhind Mathematical Papyrus, British Museum 10057 and 10058*, London: The University Press of Liverpool limited and Hodder & Stoughton limited (1923). Another version with photographs of the text was published by Arnold Chace in *The Rhind Mathematical Papyrus: Free Translation and Commentary with Selected Photographs, Translations, Transliterations and Literal Translations*, Oberlin: Mathematical Association of America (1927–29); a copy of Volume I of this is online through Wikipedia.org (accessed 8/13/2023) and it was reprinted in "Classics in Mathematics Education, Vol. 8," Reston: National Council of Teachers of Mathematics (1979). A more recent version (which is available as a Dover Publications reprint) is Gay Robins and Charles Shite, *The Rhind Mathematical Papyrus: An Ancient Egyptian Text* (1987, reprinted by Dover in 1990). Modern interpretations partition the papyrus into three parts. Part I covers arithmetic and algebra. The first part of this is a table of the values of $2/n$ for odd n ranging from 3 to 101 where the fractions are expressed as sums of "unit fractions" (that is, sums of reciprocals of integers). This is followed by a small table of fractional expressions of the numbers 1 through 9 divided by 10. As an example, 7/10 is expressed as $2/3 + 1/30$ (Eves says on his page 53 in the next section that: "The Egyptians. . . representing all fractions, except 2/3, as the sum of so-called unit fractions, or fractions with unit numerators.") These two tables seem to be evidence of an interest in division. According to Eves, the Rhind papyrus contains 85 problems (Eves' page 50), however current interpretations is that is contains 87 problems (well, 84 "Problems" and three "Numbers"; see below), four of which have a "part b." The first 40 problems cover arithmetic and elementary algebra. These problems involve multiplication, "completion" (what we would call subtraction), and simple linear equations. Part II covers geometric problems. Problems 41 through 46 concern volumes of cylindrical and rectangular granaries. Problem 47 is a table of fractional equalities related to unit conversion of volumes. Problem 48 concerns the computing of the area of a circle, including an approximation of π as $256/81 \approx 3.1605$; this is the same approximation used in Problem 41 when computing the volume of a cylinder. Problems 49 to 55 involve computations of areas rectangles, triangles, and trapezoids. The remaining problems of Book II, up to Problem 60, concern the slope (or "seked") of a pyramid. Book III covers a variety of other topics in Problems 61 to 84. These problems touch on geometric progressions, multiplication of fractions, and "pefsu" problems (concerning the strength of beer, as mentioned in Note 2.7.C on the Moscow Papyrus). The final three items, 85, 86, and 87, are referred to as "Numbers" instead of "Problems." These are written on the back side of the papyrus and are (1) a small phrase which ends the document, (2) a piece of scrap paper with words and fractions on it (this is attached to the main papyrus to help hold it together), and (3) a historical note which was probably added after the completion of the main work. Some of the flavor of the problems in the Rhind Payrus is given in Problem Studies 2.9, 2.11, 2.12, and 2.13. We'll also see more about the content of the Rhind Papyrus in the final sections of this chapter. In addition to Eves, a major source for this note is the Wikipedia webpage on the Rhind Mathematical Papyrus, which includes a description of each of the problems on the papyrus.

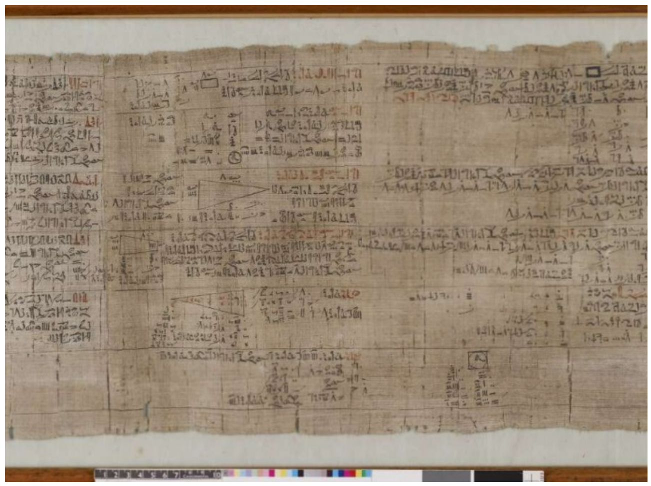
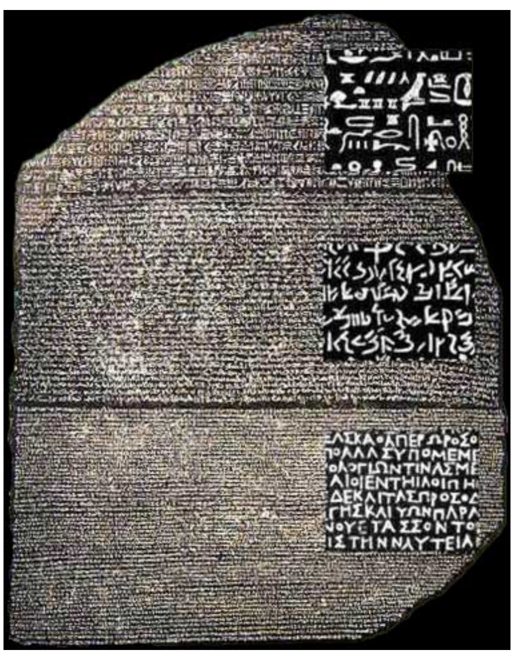


Image from The British Museum website on the Rhind Papyrus (accessed 8/11/2023)

Note. Eves mentions the "Rollin papyrus" of circa 1350 BCE, which is held the the Louvre (Eves' page 52). It is said to contain "bread accounts [presumably, recipes] showing the practical use of large numbers at the time." Unfortunately, an internet search does not turn up any additional information (or images). The Harris Papyrus of 1167 BCE is composed by Rameses IV. Written in hieratic text, it gives a summary of the entire reign of Ramesses III (between 1186 BCE and 1155 BCE; this is in apparent contradiction to Eves' date of 1167 BCE) and a listing of the temple wealth. This later list if the best example of practical accounts from ancient Egypt. It contains 1500 lines of text and is 135 feet long, making it the longest known Egyptian papyrus. This document is well-known and there is a Wikipedia webpage on Papyrus Harris I (accessed 8/11/2023), on which some of this note is based. Eves concludes his list of sources with the observation (see Eves' page 52): "Ancient Egyptian sources more recent than those just listed show no appreciable gain in either mathematical knowledge or mathematical technique."

Note. We conclude this section with the history of the deciphering of the Egyptian hieroglyphic writing. This was accomplished after the discovery the Rosetta Stone in July 1799. It was found during Napoleon's campaign in Egypt by his officer Pierre-François Bouchard. He was supervising the restoration of an old fort near the town of Rosetta (also known as Rashid, a port city in the Nile River delta 40 miles east of Alexandria) when he found a block of granodiorite (similar to granite) 3 feet 8 inches tall, 2 feet 6 inches wide, and 11 inches thick. The block, which became known as the Rosetta Stone, has three different types of writing on it. Its importance was immediately recognized. The French surrendered Egypt to the British in 1801, and the Rosetta Stone ended up in the British Museum in 1802. The three types of writing are Egyptian hieroglyphs (top), Egyptian Demotic script (middle; a script derived from forms of hieratic writing), and ancient Greek (bottom); see the image below. The first complete translation of the Greek text was published in 1803. The stone proved to be a "stele," a slab erected as a monument. The document proved to be a decree issued in Memphis, Egypt in 196 BCE for Egyptian King Ptolemy V Epiphanes written in the three versions. There are only minor differences between the three versions, allowing the stone to act as a key for deciphering the Egyptian scripts. Ancient Egyptian forms of writing, including hieroglyphic, hieratic, and demotic scripts, were no longer understood after the fourth or fifth centuries CE, since they were replaced by the

Coptic alphabet (the first alphabetic script used for the Egyptian language); see the Wikipedia webpage on "Decipherment of Ancient Egyptian Scripts" (accessed 8/12/2023). French philologist Jean-François Champollion (December 23, 1790–March 4, 1832) announced translation of the Egyptian scripts in 1822. It was discovered that both the Demotic text and the hieroglyphic text used phonetic characters to spell names and Egyptian words. This ultimately allowed the reading of the hieroglyphic engravings on ancient Egyptian structures and the beginning of the subject of Egyptology. Other multilingual inscriptions have been found of decrees dating from the early third century BCE, so the Rosetta Stone is no longer unique, but it was used in making the first steps toward understanding ancient Egyptian writing.



Images from the DiscoveringEgypt.com webpage on "Mystery of the Rosetta Stone" (left) and the Wikipedia webpage on Champollion (right). Both webpages were accessed 8/12/2023.