Chapter 3. Pythagorean Mathematics

3.1. Birth of Demonstrative Mathematics

Note. "Protohistory of humanity" is divided into three epochs: the stone age (consisting of the Paleolithic, Mesolithic, and Neolithic eras), the iron age, and the bronze age. Around 1200 BCE to 1000 BCE, the technology of producing iron (or steel) spread around the Mediterranean. With this shift in technology, came a shift in economic and military power. The civilizations of Egypt and the Babylonians waned, and the Phoenician, Greek, and Assyrian civilizations rose.

Greek Colonies, 550 B.C. from Heritage History (accessed 2/19/2022)

These new civilizations seemed to have started with trade in along the coast of Asia Minor (Turkey today) and moved west to mainland Greece, Sicily, and Italy. In mathematics, attention shifted from *how* to deal with certain concepts to *why* the concepts hold. "Thus, mathematics, in the modern sense of the word, was

born in this atmosphere of rationalism and in one of the new trading towns located on the west coast of Asia Minor." (See page 72 of Eves). The town is Miletus and it is Thales of Miletus who is credited with giving the first geometric proof, some time around 600 BCE. The traditional story is that Thales introduced logical reasoning ("proof"), instead of intuition and experiment, to the study of geometry (from which it entered mathematics in general).

Note. Thales of Miletus (circa 624 BCE–circa 547 BCE) seems to be the earliest known Greek philosopher, scientist, and mathematician. None of his writing survives and (assuming such writings ever existed) and any of them were lost by the time of Aristotle (384 BCE–322 BCE). So we have to rely on secondary sources for information on Thales, some of it written centuries after his death. For example, both Pliny (23 CE–79 CE), Plutarch (46 CE–119 CE), and Diogenes Laertius (writing in the second century CE) report that Thales used similar triangle and shadows to compute the heights of the pyramids in Egypt; this is to be addressed in Problem Study 3.1(a) and the end of this chapter. Proclus (411–485), in one of his commentaries, showed how to find the distances of ships from the shore using his theorem on similar triangles; this is to be addressed in Problem Study 3.1(b). Proclus also stated that Thales introduced the study of geometry to Greece; Eve's states that "tradition has it that demonstrative geometry began with Thales of Miletus..." (see page 72). Others are more skeptical; Bertrand Russell (1872–1970) in his *History of Western Philosophy* (1961) state that "there is no reason to believe that Thales arrived at deductive proofs, such as later Greeks discovered." However, algebraist and mathematical historian Bartel ven der Waerden (1903–1996) in his

Science Awakening (1954) claims that Thales put geometry on a logical footing and was well aware of the notion of proving a geometrical theorem. Consensus on the contributions of Thales to geometry 2600 years ago is not likely to be reached! Thales is considered one of the "seven sages" of Greece (or "seven wise men"), a list first mentioned in Plato's (circa 425 BCE–circa 347 BCE) *Protagoras* (Socrates is quoted as including Thales in the list of seven "perfectly educated men" with a "love of wisdom").

Thales of Miletus (624 BCE–547 BCE)

This biographical information and the image of the Thales stamp is from the MacTutor History of Mathematics Archive biography of Thales (accessed 7/23/2021). Another reference for biographical information on Thales is D. R. Dicks' "Thales," *The Classical Quarterly, New Series*, **9**(2), 294–309 (Nov., 1959). This can be viewed online on the JSTOR website (accessed 2/7/2023).

Note. Thales is credited with five geometry results:

1. A circle is bisected by any diameter.
2. The base angles of an isosceles triangle are equal.
3. The vertical angles formed by two intersecting lines are equal.
4. Two triangles are congruent if they have two angles and one side equal.
5. An angle inscribed in a semicircle is a right angle.

The MacTutor Thales biography webpage gives some references as to why these five results are commonly associated with Thales. It states that Proclus (writing around 450 CE) is the source for the first four claims. Proclus quotes Eudemus of Rhodes' *History of Geometry* (Eudemus was Aristotle's student) for the third and fourth claims; Eudemus's book is lost and, regrettably, only indirect references to it exist today. The fifth claim is based on a Diogenes Laertius's *Lives of the Eminent Philosophers* written in the second century CE. The absence of original work by Thales and the secondary nature of references to his accomplishments has to be considered when trying to assess the real accomplishments of Thales. We'll see in the next next section that a similar situation exists when dealing with Pythagoras.

Note. In connection with the fourth claim, "The vertical angles formed by two intersecting lines are equal," we consider Figure 6 (below). We want to show that angle a is equal to angle b. The result is not surprising and one might imagine rigidly moving one angle until it is seen to coincide with the other angle. By introducing angle c, we see that angle a plus angle c equals a "straight angle" (a

straight angle is a 180° angle). Similarly, angle b plus angle c equals a straight angle. Quantitatively, we have $a + c = 180°$ and $b + c = 180°$, so that (since "all straight angles are equal") angle a equals angle b. That is $a + c = b + c$, and quantitatively "if equals are subtracted from equals, the remainders are equal," so that $(a + c) - c = (b + c) - c$ or $a = b$.

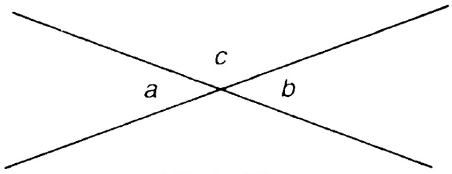

FIGURE 6

Note. Thales is credited with work in areas other than mathematics. In a study on navigation, he is said to have defined the constellation Ursa Minor (which includes the north star). Herodotus (484 BCE– circa 425 BCE) in his *The Histories* (1.73-74) claimed that Thales foretold the May 28, 585 BCE solar eclipse, though this is very with skepticism today. Totality was visible along the northern Mediterranean, and this is sometimes called the "Eclipse of Thales" (see the Wikipedia page on the Eclipse of Thales). The philosophy of Thales is known from Aristotle's (385 BCE–322 BCE) *Metaphysics*, in which he explains that Thales thought the Earth was a flat disc floating on an infinite ocean and that earthquakes are explained by the fact that the Earth floats on water. As with his introduction of rigor and proof to geometry, the importance of this idea is that it makes Thales the first recorded person who tried to explain a natural phenomena by rational means, without an appeal to the supernatural. (This information is also from the MacTutor biography of Thales.)