3.10. Postulational Thinking

Note. In this section, we compare the ideas of "axiom," "postulate," and "hypothesis." We quote Aristotle's *Posterior Analytics* to give the thoughts on these ideas at the time of Euclid.

Note. We start by reproducing most of Eves' (very brief) Section 3-10:

> "Sometime between Thales in 600 B.C. and Euclid in 300 B.C., the notion was perfected of a logical discourse as a sequence of rigorous deductions from some initial and explicitly stated assumptions. This process, the so-called **postulational method**, has become the very core of modern mathematics; undoubtedly, much of the development of geometry along this pattern is due to Pythagoreans. Certainly one of the greatest contributions of the early Greeks was the development of this postulational method of thinking."

Note. In contemporary terminology, the terms "axiom" and "postulate" are interchangeable. This is briefly discussed in my online notes for Introduction to Modern Geometry (MATH 4157/5157), the history component, on Section 1.3. Axiomatic Systems. In the *Elements*, Euclid states five postulates and five axioms (which he calls "common notions"). Thomas Heath in his *The Thirteen Books of Euclid's Elements Volume I*, in Chapter IX, section 3 ("First Principles: Definitions, Postulates, and Axioms"), extensively quotes Aristotle's (384 BCE–322 BCE) *Posterior Analytics*. Aristotle's comments include (see Heath's pages 117–119):

> "By first principles in each genus I mean those the truth of which it is not possible to prove. ...Now anything that the teacher assumes...without proving it himself, is a hypothesis if the thing assumed is believed by the learner, and it is moreover a hypothesis, not absolutely, but relatively to the particular pupil; but, if the same thing is assumed when the learner either has no opinion on the subject or is of a contrary opinion, it is a postulate. This is the difference between a hypothesis and a postulate. ...A hypothesis is that from the truth of which, if assumed, a conclusion can be established.

Heath continues on his page 119: "Every demonstrative science, says Aristotle, must start from indemonstrable principles: otherwise, the steps of demonstration would be endless. Of these indemonstrable principles some are (*a*) common to all sciences, others are (*b*) particular, or peculiar to the particular science; (*a*) the common principles are the *axioms*." Aristotle's views reflect the state of the postulational method as it stood following its evolution from Thales, through the Pythagoreans, to the time of Aristotle and Euclid.

Note. A modern (i.e., post David Hilbert [January 23, 1862–February 14, 1943]) approach to an axiomatic development of geometry is given in Introduction to Modern Geometry (MATH 4157/5157); I have online notes for this class at Introduction to Modern Geometry—Axiomatic Method class notes. The need for undefined terms, as Aristotle also mentions in *Posterior Analytics*, is explored in Section 1.3. Axiomatic Systems of these notes.

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