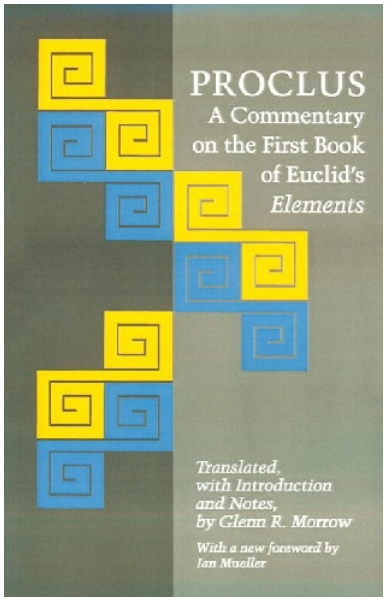
3.2. Pythagoras and the Pythagoreans

Note. A common sentiment among historians of mathematics is that Euclid's *Elements* was so comprehensive that it made previous work in Greek mathematics obsolete, ultimately leading to the loss of the earlier works. As Eves puts it (see page 74): "The history of the first 300 years of Greek mathematics is obscured by the greatness of Euclid's *Elements*, written about 300 B.C., because this work so completely eclipsed so many preceding Greek writings on mathematics that those earlier works were thenceforth discarded and have become lost to us." As a result, evidence of pre-Euclid Greek mathematics is very largely based on second-hand sources. Using these sources (some written hundreds of years after the fact), scholars of classicism have constructed a "rather consistent, although somewhat hypothetical, account of the history of early Greek mathematics..." [Eves, page 74]. Eves credits five such scholars, but two will play very prominent roles. Johan Ludwig Heiberg (1854–1928) translated many classical Greek works of mathematics and astronomy into German. More influential for us is Sir Thomas Little Heath ((1861–1940), who translated much of Heiberg's work into English (including a widely circulated three volume version of Euclid's *Elements*, which is still in print by Dover Publications).

Note. The gap in knowledge of Greek mathematics before Euclid is filled in part by Proclus Diadochus' (circa 411–April 17, 485) commentary on Euclid's *Elements*. Eves refers to this as the *Eudemian Summary* because Proclus is reporting on a history written by Eudemus of Rhodes (circa 350 BCE–290 BCE). Proclus includes

this in his commentary on Book I of Euclid's *Elements*. This commentary is still in print today as *Proclus: A Commentary on the First Book of Euclids Elements*, translated by Glenn R. Morrow (Princeton University Press, 1970; a 1992 version with a new forward by Ian Mueller is also available). This is the source of our information on Thales, and a number of other mathematicians who are only known from this source. A fairly thorough description of the content of Proclus' summary of Eudemus' work is given in supplement to these class notes on Supplement. Proclus's Commentary on Eudemus' *History of Geometry*.



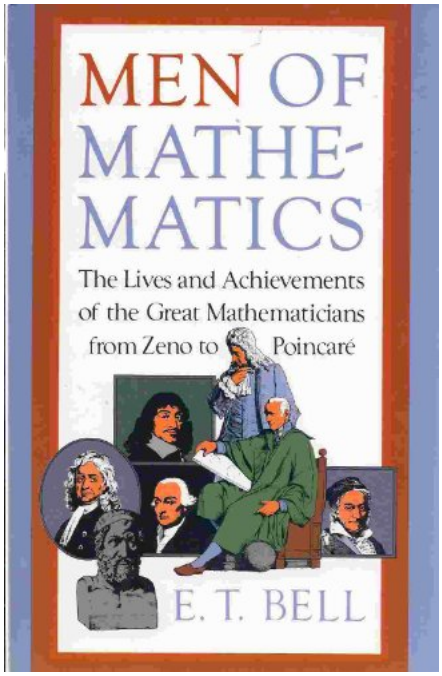
Note. We now turn our attention to Pythagoras of Samos (circa 570 BCE–circa 490 BCE). In E. T. Bell's *Men of Mathematics* (Simon and Schuster, 1937), Pythagoras is described as follows (on his page 20): "His life has become fable, rich with the incredible accretions of his prodigies; but only this much is of importance for the development of mathematics as distinguished from the bizarre number-mysticism in which he clothed his cosmic speculations: he travelled extensively in Egypt,

learned much from the priests and believed more, visited Babylon and repeated his Egyptian experiences; founded a secret Brotherhood for high mathematical thinking and nonsensical physical, mental, moral, and ethical speculation at Croton in southern Italy; and, out of all this, made two of the greatest contributions to mathematics in it entire history. He died, according to one legend, in the flames of his won school fired by political and religious bigots who stirred up the masses to protest against the enlightenment which Pythagoras sought to bring them."

This is a close-up of Raphael's *The School of Athens* (painted between 1509 and 1511, located in the Apostolic Palace in Vatican City) showing his depiction of Pythagoras; from the Wikipedia page on Pythagoreanism (accessed 2/18/2023).

Note 3.2.A. E. T. Bell's *Men of Mathematics* is quoted in the previous note. This is an early popular-level book on the history of mathematics. It is an interesting read, but the history that it contains should be approached with a degree of skepticism and/or caution. For example, on the Wikipedia page for *Men of Mathematics*, C. Tresdell (in his 1984 "Genius and the establishment at a polite

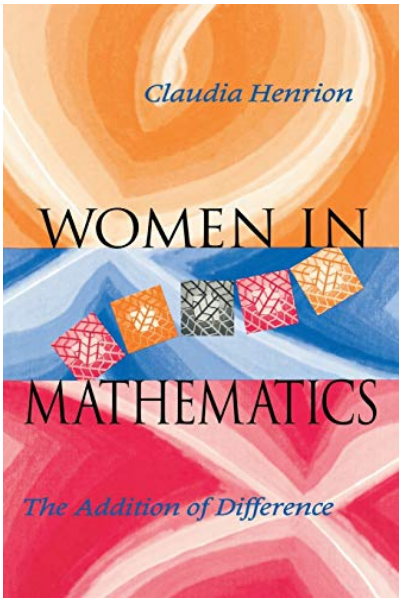
standstill in the modern university: Bateman," in *An idiot's fugitive essays on science: methods, criticism, training, circumstances.* Berlin: Springer-Verlag, pages 423–424) is quoted as: "... [Bell] was admired for his science fiction and his *Men of Mathematics*. I was shocked when, just a few years later, Walter Pitts told me the latter was nothing but a string of Hollywood scenarios; my own subsequent study of the sources has shown me that Pitts was right, and I now find the contents of that still popular book to be little more than rehashes enlivened by nasty gossip and banal or indecent fancy."



The title reflects an obvious sexist leaning. The Wikipedia page also states that Bell did not like the title given to the book by the publisher. His contract with Simon and Schuster was for a book titled *The Lives of Mathematicians*, but it was changed to *Men of Mathematics* by Simon and Schuster to tie it to another book they had published, *Men of Art* by Thomas Craven (this detail is from Constance Reid's *The Search for E. T. Bell: Also Known as John Taine*, Mathematical Association of

America, 1993). Bell's book is not devoid of the mention of women mathematicians (though there is not chapter devoted a woman); there is a brief mention of Sophie Germain, Sonja Kowalewski, and Emmy Noether.

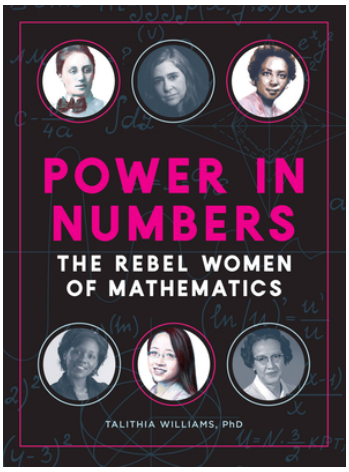
Note 3.2.B. There are apparently two books with the title *Women in Mathematics*. The first if by Lynn Osen, copyright 1974 by The Massachusetts Institute of Technology. This book is similar in structure to Bell's book and includes chapters on Hypatia, Maria Agnesi, Emilie de Breteuil (du Châtelet), Caroline Herschel, Sophie Germain, Mary Fairfax Somerville, Sonya Kovelevsky, and Emmy Noether. The second book is by Claudia Henrion and titled *Women in Mathematics: The Addition of Difference* (Indiana University Press, 1997). This appears in a series on race, gender, and science. Its focus is more on sociology than the history of mathematics, and it addresses (with first-hand accounts) the experiences of women, people of color, and the "aged" (which falls, in this book, in the category of non-"youth"!) in their careers in mathematics, mostly in the university environment.



Note 3.2.C. This is an appropriate place to mention "Women in Mathematics," an organization based at the Institute for Advanced Study (at Princeton University) supported, in part, by the National Science Foundation. Their Mission webpage states: "The Women and Mathematics Program (WAM) at the Institute for Advanced Study is an annual program with the mission to recruit and retain more women in mathematics. WAM aims to counter the initial imbalance in the numbers of men and women entering mathematics training as well as the higher attrition rate of female mathematicians compared to their male counterparts at every critical transition stage in mathematical careers." More information is on the Women and Mathematics (WAM) webpage.

Note 3.2.D. A newer publication along these lines is Talithia Williams' *Power in Numbers: The Rebel Women of Mathematics* (Race Point Publishing, 2018). Less technical than Osen's book, this source contains biographies through three parts, pre-twentieth century (in Part I: The Pioneers), applications in the first half of the twentieth century (Part II: From Code Breaking to Rocket Science), and beyond the mid twentieth century ((Part III: Modern Math Mavens). Biographies not readily available in other general sources include those of Grace Hopper (December 9, 1906–January 1, 1992; a computer pioneer who did Ph.D. work in algebra at Yale), Katherine Johnson (August 26, 1918–February 24, 2020; 2015 Presidential Medal of Freedom recipient and lead charcter in the 2016 film *Hidden Figures* about the by-hand computations done for the first U.S. manned orbital flight of John Glenn; she was the first African-American female admitted to graduate school at the University of West Virginia, and she taught in Marion, VA [which is about 80

miles up Interstate 81 from Johnson City, TN)), and Ingred Daubechies (August 17, 1954– ; a pioneer in wavelet theory and image compression). *Power in Numbers* is meant to tell inspirational stories to motivate readers to undertake the study of math and science. On page 9 of the Introduction, Williams states: "I invite women and men of all nationalities and backgrounds to learn about these dynamic mathematicians and scientists that have shaped out society. May their stories empower the next generation of STEM rebels to continues advancing mathematical theory, bringing awareness to the field, and increasing our Power in Numbers." Recently, Williams hosted an episode of the PBS series NOVA titled "Zero to Infinity" (premiered November 16, 2022).



On the right is a scene from the NOVA program "Zero to Infinity" (2022), presented by Dr. Talithia Williams.

Note. Now back to Pythagoras. It seems that E. T. Bell's brief description of Pythagoras given above is broadly accepted as factual. No writings by Pythagoras or the Pythagoreans (that is, the members of the "secret Brotherhood" referred to by Bell) are known, nor did they likely ever exist given the secret nature of the Pythagorean school/brotherhood. Some of the earliest known references to

Pythagoras seem to be due to Aristotle (384 BCE–322 BCE) in his *Metaphysics*. Pythagoras is also mentioned in Eudemus' (circa 350 BCE–290 BCE) *History of Geometry*, now lost but summarized by Proclus Diadochus (circa 411 CE–April 17, 485 CE) in his commentary on Euclid's Book I: "...Pythagoras transformed mathematical philosophy into a scheme of liberal education, surveying its principles from the highest downwards and investigating its theorems in an immaterial and intellectual manner. He it was who discovered the doctrine of proportionals and the structure of the cosmic figures." See page 53 of *Proclus: A Commentary on the First Book of Euclid's Elements*, translated by Glenn R. Morrow (Princeton University Press, 1970) and Supplement. Proclus's Commentary on Eudemus' *History of Geometry* (Proclus Diadochus lived circa 411–April 17, 485). The philosophy/religion of Pythagoras, "Pythagoreanism," is described in detail on the Stanford Encyclopedia of Philosophy webpage (accessed 2/22/2023). Four biographies of Pythagoras survive, though only one of them dates from the *bce* time. They are *The Life of Pythagoras* by Diogenes Laertius (3rd century BCE), *Life of Pythagoras* by Porphyry of Tyre (233 CE–309 CE), *The Life of Pythagoras* by Iamblichus of Chalcis (circa 250 CE–circa 330 CE), and *The Anonymous Life of Pythagoras* preserved by Photius (circa 820-891). These appear in *The Pythagorean Sourcebook and Library: An Anthology of Ancient Writings Which Relate to Pythagoras and Pythagorean Philosophy* complied and translated by Kenneth S. Guthrie (Phanes Press, 1987; originally published in 1920). The longest of the biographies is that of Iamblichus, which is about 65 pages. The other three range from to 17 pages in length. These biographies are more concerned with the Pythagorean philosophy and religion, than with math. Some of the mystical religious ideas of Pythagoras

live on today in the new age topics of numerology and sacred geometry (Wikipedia will have information on this).

Note. To give a flavor of Pythagorean mysticism in connection with numbers, we quote from Mario Livio's *Is God a Mathematician?* (Simon and Schuster, 2009). This is a nice history of some of the philosophy of math, concentrating on Platonism (in spite of the provocative title). Livio states (on pages 234 and 235):

> "...the Pythagoreans were obsessed with numbers. They thought of the odd numbers as being masculine and good, and, rather prejudicially, of the even numbers as being feminine and bad. They had a particular affinity for the number 5, the union of 2 and 3, the first even (female) and first odd (masculine) numbers. (The number 1 was not considered to be a number, but rather the generator of all numbers.) To the Pythagoreans, therefore, the number 5 portrayed love and marriage, and they used the pentagram—the five-pointed star...—as the symbol of their brotherhood. Here is where the golden ratio makes its first appearance. ...In fact, to construct a pentagon using a straight edge and a compass (the common geometrical construction process of the ancient Greeks) requires dividing a line into the golden ratio.

The golden ratio and pentagons take on additional physical/mystical importance when Plato associates the five platonic solids with the (Greek) elements of the universe. There is more about this in Section 3.9. The Regular Solids (see also Problem 3.14). The golden ratio is also addressed in the history component of Introduction to Modern Geometry (MATH 4157/5157) in Section 1.4. The Regular Pentagon.

Note. We turn to Jason Socrates Bardi's *The Fifth Postulate: How Unraveling a Two-Thousand-Year-Old Mystery Unraveled the Universe* (Hoboken, NJ: John Wiley & Sons, 2009) for a summary of Pythagoras' life.

> "Pythagoras was born around 580 BC on the ancient island city of Samos, off today's Turkish coast. He grew up in a culture that was thriving at the dawn of Greek civilization in the sixth century BC. . . . He studied under some of the greatest teachers of his day. . . He may even have studied under Thales and Anaximander. . . In the Near East Pythagoras lived among Arabian and Jewish communities. He also lived in Babylon and spent a long time in Egypt. . . left Samos and settled in the city of Croton in the Calabria region of southern Italy where he established his popular school around 530 BC. He was over sixty at the time. Pythagoras's arrival in Italy was a step in the history of geometry because of the way he pushed mathematics on his followers." (pages 32 and 33)
>
> "Pythagoras's school sounds more like a cult than anything else in our experience today. . . It is not known whether it was Pythagoras or his disciples who invented music theory. He originated the idea that musical intervals can be expressed as numerical intervals. . . So influential was Pythagoras's work in music that centuries later Plato considered music theory equal in importance to astronomy. . . . He avoided wine and meat, and he absolutely forbade the eating of hearts. One of his most famous dietary restrictions was on beans or lentils. In fact, all foods that caused flatulence were forbidden." (pages 35 and 36)

Bardi turns to Pythagoras' contributions to math and describes his death as follows.

> "Pythagoras, abstracted by veils himself, was the first person to really put abstract reasoning into mathematics. He set geometry on a path from which it would never turn back... the Pythagoreans were at heart a mysterious cult that upheld a common pact not to disclose the revelation of mysteries that bound them together. ... His end came about as a sort of backlash against him and his cult. ... One such plotter was a wealthy and powerful man named Cylon, who was from Croton... Cylon was of the most privileged class... Cylon saw it as his right to be accepted into the Pythagorean fold... [Pythagoras] gave no special consideration to Cylon. Spurned, Cylon started a conspiracy to oppose Pythagoras, which led to what have been called the Pythagorean riots. ... The plotters succeeded in stirring up enough resentment toward Pythagoras... The mob set upon the house with stones and torches, burning it and stoning the Pythagoreans inside. ... Once free of the burning building, Pythagoras was chased from town to town by the crowds until he finally found refuge at a temple where, according to one story, he slowly starved to death over the course of several weeks. Another account says he died from grief, not lack of food." (pages 37, 38, and 39)

Note. Since these are history of *math* notes, we shift from the personal details and mystical ideas of Pythagoras to the mathematical contributions of Pythagoras (or at those contributions which are, by tradition, attributed to him). In Sections 3.3 to 3.7 we consider several mathematical topics which are often attributed Pythagoras

or the Pythagoreans. Since the historical record is scant for this period of time, we often rely on Euclid's *Elements* for material of that era. It can be safely assumed that Euclid was influenced by Pythagoras (and other predecessors) so that we can reasonably rely on the *Elements* as a "stand-in" for original versions of Pythagoras' work.

Revised: 3/10/2023