# 4.2. Lines of Mathematical Development

**Note.** In this short section we consider three separate lines of mathematical development during the 300 years leading up to Euclid's *Elements*. We mention some of the key characters in each line and mention where they are discussed in this course.

**Note.** The first line of development concerns the material that ultimately became Euclid's *Elements*. Pythagoras (circa 570 BCE–circa 490 BCE) and the Pythagoreans were the topic of the previous chapter, and we often mentioned where their ideas appeared in the *Elements*. Hippocrates (circa 470 BCE–circa 410 BCE), mentioned in Supplement. Proclus's Commentary on Eudemus' *History of Geometry*, found areas of lunes using the Pythagorean Theorem (for details, see my online notes for Introduction to Modern Geometry [MATH 4157/5157] on Section 1.8. Three Famous Problems of Greek Geometry) and wrote his own elements of geometry book. Hippocrates also reduced the doubling the cube to consideration of certain proportionals, as will be discussed in Section 4.5. Duplication of the Cube. As discussed in Section 3.5. Discovery of Irrational Magnitudes, Theodorus (465 BCE–398 BCE) constructed $\sqrt{n}$ for $3 \leq n \leq 17$ (see also Problem 3.15). Theaetetus (circa 417 BCE–circa 369 BCE) developed a theory of irrationals which appears in Euclid's Book X, and constructions of the five regular solids which appear in Euclid's Book XIII. Eudoxus (408 BCE–355 BCE) contributed to the theory of proportions which appears in Euclid's Books V and VI and may have been the first to use the method of exhaustion. For both Theaetetus and Eudoxus, see also Supplement. Proclus's Commentary on Eudemus' *History of Geometry*).

**Note.** The second line of development concerns notions connected with infinitesimals, limits, sequences, and series; that is, with the very early conceptual ideas that lead to calculus. Zeno of Elea (490 BCE–425 BCE) and his paradoxes are introduced in Section 11.2. Zeno's Paradoxes. The method of exhaustion of Antiphon the Sophist (480 BCE–411 BCE) and Eudoxus (408 BCE–355 BCE) is presented in Section 11.3. Eudoxus' Method of Exhaustion. Democritus of Abdera (circa 460 BCE–circa 370 BCE), his atomistic theory, and his idea of a solid as an infinite sum of plane parts will be seen in Chapter 11, "The Calculus and Related Concepts."

**Note.** The third line of development is that of higher geometry. This includes curves other than circles and lines, and other than spheres and planes. Examples are conic sections and cones. Much of this line of development came about in attempts to solve the "three famous [compass and straight edge] constructions." These ideas are covered in the remainder of this chapter.

*Revised: 3/13/2023*