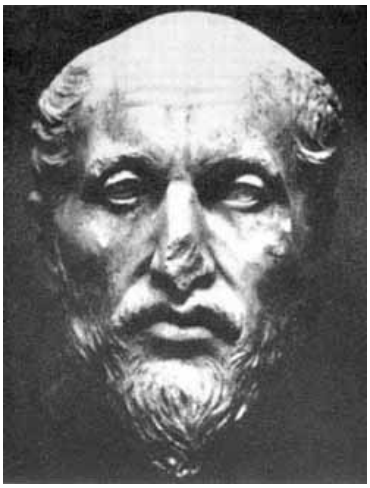
5.3. Euclid's "Elements"

Note. In this section we consider the history of the most influential book on mathematics! The axiom/definition/theorem/proof approach of the *Elements* persists in every upper-level math book to this day. Heath in his translation of the *Element's* states (see his page vii): "It is one of the noblest monuments of antiquity; no mathematician worthy of the name can afford not to know Euclid, the real Euclid as distinct from any revised of rewritten versions which will serve for school [children] or engineers." We consider Greek commentaries on the *Elements* with special attention to Proclus (circa 411 CE–April 17, 485 CE), some of the Arabic translations from the 9th through the 13th century, other translations from the 9th through the 12th century, the first printed version, and English translations from the 16th through the 18th century. The most widely used version of the *Elements* today is Thomas Heath's *The Thirteen Books of Euclid's Elements*, Translated from the Text of Heiberg, with Introduction and Commentary, Volumes I, II, and III, Cambridge University Press (1908; a second edition of Volume I was published in 1925). We give background on this version, on Sir Thomas Heath, and on Johan Ludvig Heiberg. In fact, much of this section of notes is based on the 150 page Introduction of Heath's book. When referencing this Introduction, we simply refer to it as "Heath, page..." Finally, we consider the *Elements* in the 21st century.

Note. Eves starts this section with the following description of the impact of the *Elements* (see page 141):

> "It appears that this remarkable work immediately and completely superseded all previous *Elements*; in fact, no trace remains of the earlier efforts. As soon as the work appeared, it was accorded the highest respect, and from Euclid's successors on up to modern times, the mere citation of Euclid's book and proposition numbers was regarded as sufficient to identify a particular theorem or construction. No work except the Bible, has been more widely used, edited, or studied, and probably no work has exercised a greater influence on scientific thinking. Over one thousand editions of Euclid's *Elements* have appeared since the first one printed in 1482; for more than two millennia, this work has dominated all teaching of geometry."

Note 5.3.A. No copy of Euclid's *Elements* exist from his time. So our knowledge of the original *Elements* of Euclid are based on secondary sources, primarily commentaries by Theon of Alexandria (circa 335 CE–circa 405 CE), Proclus Diadochus (circa 411 CE–April 17, 485 CE), and the 10th century "Vatican Manuscript #190."



Images from the MacTutor biography webpages on Heron of Alexandria (left) and Proclus Diadochus (right).

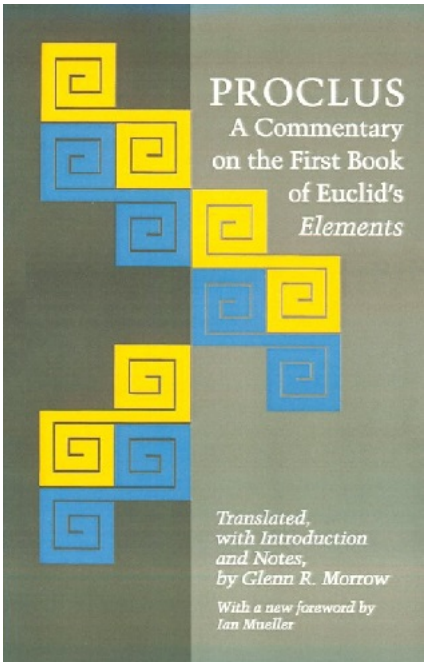
Heron of Alexandria (sometimes "Hero"; circa 10 CE–circa 75 CE), who we consider in more detail in Section 6.6. Heron, is known to have written a commentary on Euclid's *Elements*. Only fragments of this work survive, but from these it is known that it covered at least eight books in the *Elements* (according to the MacTutor biography webpage on Heron of Alexandria; accessed 7/16/2023). Theon of Alexandria (notice that he lived about 700 years after Euclid) was the only Greek text of the *Elements* known until the discovery of Vatican Manuscript #190 in the late 1800s. It seems that Theon tries to improve on an earlier manuscript by correcting mistakes he find. However, sometimes he makes changed thinking that he is correcting mistakes when, in fact, he fails to deeply understand the material. In addition, he added details whenever he though the Euclid's arguments were too brief (including adding new propositions). Theon's edition became the standard edition of the *Elements* and almost all earlier editions were lost. It is through that Theon wrote with the assistance of his daughter, Hypatia (who we consider in more detail in 6.10. The Commentators and Supplement. Hypatia of Alexandria). This information on Theon is based on the MacTutor biography webpage on Theon of Alexandria; accessed 7/16/2023). The reason we know that Theon has made editorial changes is because of the discovery by math historian François Peyrard in 1808 of a book in the Vatican holdings. It contains, the *Elements* Books I-XIII, with scholia (or "explanatory comments"), a commentary on the *Data* and related scholia, then the (so-called) *Elements* Books XIV and XV. Book XIV extends Euclid's Book XIII and considers regular solids inscribed in a sphere; it was written by Hypsicles of Alexandria (cica 190 BCE–circa 120 BCE). Book XV covers regular solids and is believed to have been written by Isidore of Miletus (circa 442 CE–537

CE); see Frank Swetz and Victor Katz's "Mathematical Treasures—Christopher Clavius's Edition of Euclid's *Elements*" webpage (accessed 7/16/2023). Unlike the other existing commentaries, it lacks the editorial changes made by Theon, making it of unique historical interest and providing us with the closest thing available to Euclid's original work. This is the main source used by Johan Heiberg when developing his German translation on which Thomas Heath's English translation is based. The book can be viewed online on the Digital Vatican Library, DIGIVATLIB, or DVL (accessed 7/16/2023); is was first posted here on February 15, 2016.

Image of the page of Vatican Manuscript #190 containing the Pythagorean Theorem. From ibiblio, The Public's Library and Digital Archive (accessed 7/16/2023).

Note 5.3.B. Proclus Diadochus (or Proclus Lycius; circa 411 CE–April 17, 485 CE) provides us with one of the two main sources of information as to the history of Greek geometry which we possess, the other being the *Collection* of Pappus (circa

290 CE–circa 350 CE; see Note 5.3.C); this according to Heath in his *Elements* on page 29. Proclus work is his *Commentary* on Book I of Euclid's Elements. It is currently in print as *Proclus: A Commentary on the First Book of Euclid's Elements*, Translated with Introduction and Notes by Glenn Morrow (Princeton University Press, 1970).

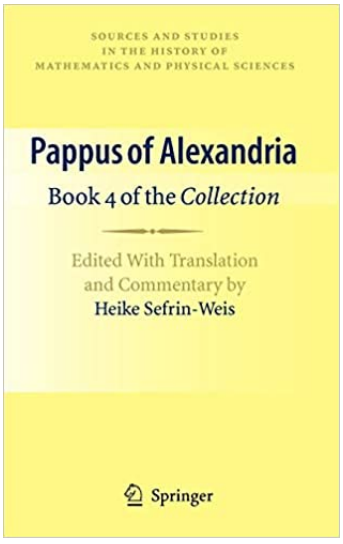


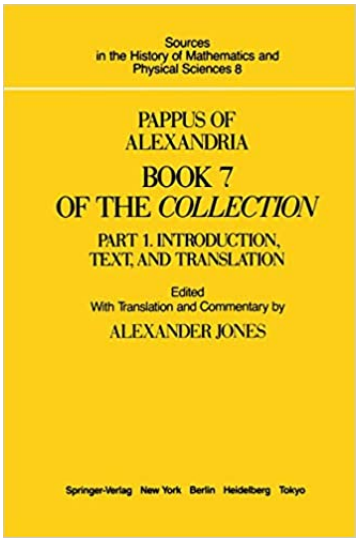
A pupil of Proclus, Marinus, wrote *Vita Procli* ("The Life of Proclus") which survives and gives us a rather detailed story of his life. Proclus was born in Byzantium (in modern-day Turkey). He received his early education in the coastal town of Xanthus. His main interest was philosophy, and he studied the philosophy of Aristotle and the mathematics of Heron in Alexandria. He went to Athens and joined the School of Plato, where he would spend his entire later career. He became a prolific writer early in his career and ultimately produced a huge body of writing. He wrote on topics of including philosophy, theology, mathematics, astronomy, physics (as it was in his time), literary criticism, and poetry. Some of his philosophical writings were presented in the fashion of geometry in that it

followed in the proposition/proof style. A similar-style work is Benedict Spinoza's (November 24, 1632–February 21, 1677) *Ethics, Demonstrate in Geometrical Order* published posthumously in 1677. With his interest in philosophy, his mathematical commentaries often are interrupted by digressions related to the moral or metaphysical significance of a theorem or geometric figure. He was the last creative mind in Greek philosophy. Less than 50 years after his death, the pagan philosophies and religions, to which Proclus subscribed, were stilled in Athens by the Byzantine emperor. These historical notes on Proclus are based on Morrow's translation of Proclus' *Commentary on Book I*, pages xxxix–xlv. As stated in Supplement. Proclus's Commentary on Eudemus' *History of Geometry*, Proclus's *Commentary on Book I* also includes most of what we know about pre-Euclidean Greek geometry.

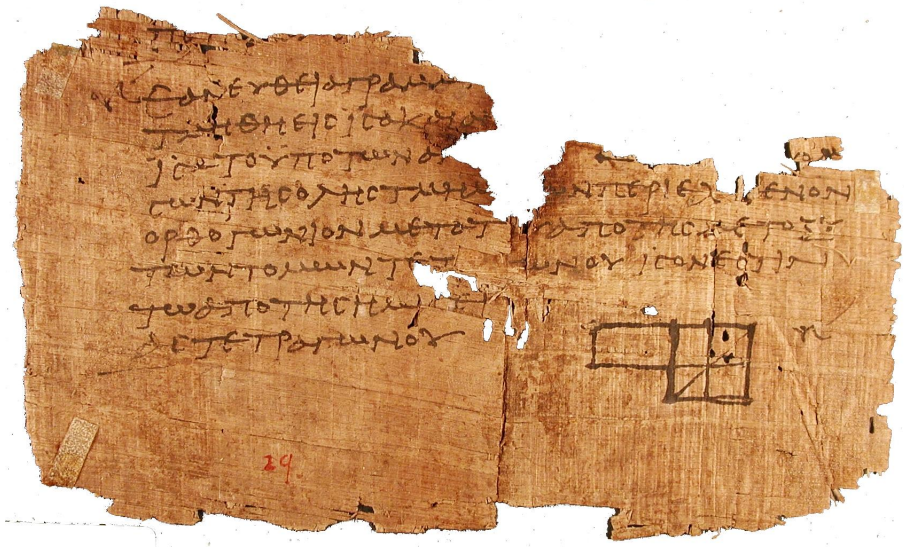
Note 5.3.C. Pappus of Alexandria (circa 290 CE–circa 350 CE) is "the last of the great Greek geometers" according to the MacTutor History of Mathematics Archive biography of Pappus (from which the following history is based). His *Mathematical Collection* (or *Synagoge*), written around 340 CE, consists of eight books. Though this does not exclusively address Euclid's *Elements*, it is one of the two main sources of information as to the history of Greek geometry which we possess (the other being Proclus' *Proclus: A Commentary on the First Book of Euclid's Elements* of Note 5.3.B). Book I is now lost, but it covered arithmetic. Book II is partly lost but the remaining part deals with Apollonius's method for dealing with large numbers. Book III deals with constructing means, geometrical paradoxes, and inscribing polyhedra in spheres. Book IV contains properties of curves including the spiral of Archimedes and the quadratrix of Hippias,and addresses

using these to trisect an angle. More details on this can be found in my online notes on the historical component of Introduction to Modern Geometry (MATH 4157/5157) on Section 4.1. The Conchoid of Nicomedes, The Trisection of an Angle (the material of those notes heavily overlap with this note). In fact, Book IV is still in print as *Pappus of Alexandria: Book 4 of the Collection*, Edited With Translation and Commentary by Heike Sefrin-Weis, Sources and Studies in the History of Mathematics and Physical Sciences (Springer, 2010). Book V describes hexagonal bee honeycombs and discusses the thirteen semiregular solids of Archimedes. Book VI deals with books on astronomy by others. Book VII considers the "Treasury of Analysis" and focuses on the work of Euclid, Apollonius, and Aristaeus the elder. Part 1 of this is in print as *Pappus of Alexandria Book 7 of the Collection: Part 1. Introduction, Text, and Translation* Edited and translated by Alexander Jones, Sources in the History of Mathematics and Physical Sciences 8 (softcover reprint of the original 1st edition, Springer-Verlag, 1986 Edition). Book VIII deals with mechanics (movement of bodies about their centers of gravity, motion in space, and forces/collisions).

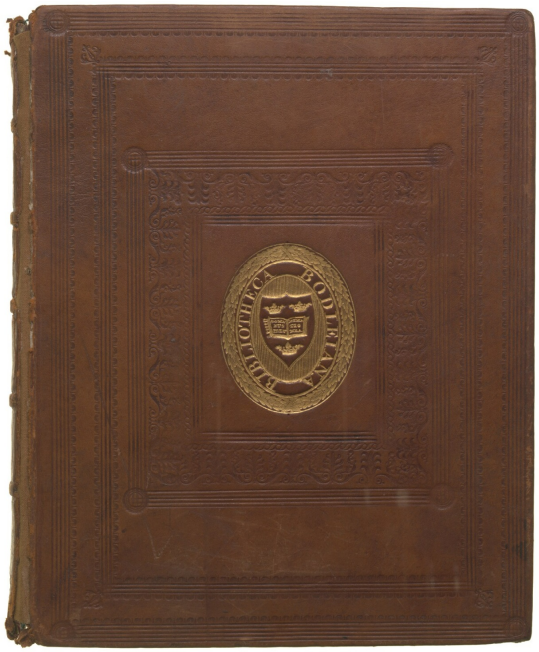




Note 5.3.D. Oxyrynchus (modern day Al-Bahnasa) is located in Egypt on the Nile River, about 100 miles south of Cairo. Starting in 1896 two archaeologists, Bernard Grenfell and Arthur Hunt of Queens College Oxford, excavated rubbish piles in Oxyrynchus. They discovered a treasure trove of papyrus fragments. One of the fragments, denoted "Papyrus Oxyrhynchus 29," contains the oldest diagram from Euclid's *Elements*; it dates from 75–125 CE. It concerns Proposition 5 of Book II of the *Elements* (which states, as Heath translates it: "If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half."). This is an example of geometric algebra (introduced in Section 3.6. Algebraic Identities) and represents the identity $ab + (a-b)^2/4 = (a+b)^2/4$. This history and the following image are from Bill Casselman's, of the University of British Columbia, "One of the oldest extant diagrams from Euclid" webpage (accessed 7/17/2023). By the way, the Oxyrynchus site has since resulted in over 5,000 reconstructed documents.



Note 5.3.E. We now consider several other non-Arabic middle ages versions of the *Elements*. Each incorporates the editorial changes of Theon, as described in Note 5.3.A. The Bodleian manuscript dates from 888 CE. It is currently in the Bodleian Library at the University of Oxford as manuscript "MS D'Orville 301." This manuscript contains Books I to XV of the Elements with many scholia. It was copied (in Greek on parchment) by Stephen the Clerk for Arethas of Patras in Constantinople in 888 and, according to the Bodleian Library exhibition catalogue, *The Survival of Greek Literature*, it is the oldest manuscript of a classical Greek author to bear a date. I has been digitized and can be viewed online at the Digital Bodleian webpage (the source of the following image; accessed 7/17/2023).



Other versions of the *Elements* used by Johan Heiberg in the preparation of his German translation (which, in turn, was used by Heath in the preparation of his English translation) include:

1. Manuscript XXXVIII, 3 of the Laurentian Library in Florence, Italy which dates

from the 10th century, includes Books I-XV, *Optics*, and *Phaenomena* (both also by Euclid).The original writing has been renewed in places and many repairs and additions have been made.

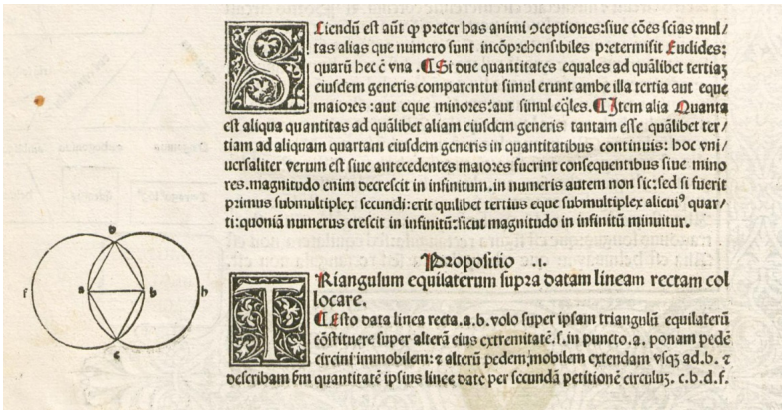
2. Manuscripts 18 and 19 of the Communal Library in Bologna, Italy from the 11th century includes Books I-XIII and the *Data* (also by Euclid). The margins include scholia written by the original scribe, and two or three other later hands. It includes the definitions and propositions of the *Elements* for Books I–XIII. Heiberg conjectures that the manuscript is due to Byzantine mathematician who though Euclid's proofs were to long and complicated. This manuscript reveals that the Byzantine scribe likely had a pre-Theon manuscript available from which he copied.

3. The Viennese Manuscript ("Philos. Gr. No. 103) probably from the 12th century, which includes Books I–XV, *Optics*, and *Phaenomena*. It appears to be written by several different scribes. Though bounded into a single volume, it appears to have originally been in two volumes.

4. Two Paris Manuscripts from the 12th century. The first, Paris Manuscript 2466, is written in two hands and includes Books I–XIII of the *Elements* with scholia after Books XI, XII, and XIII. The second, Paris Manuscript 2344, contains Books I–XIII of the *Elements*, all written in the same hand, with scholia by many hands.

The first printed version of *The Elements* appeared in May 1482 in Venice published by Erhard Ratdolt. The text was based upon a translation from Arabic to Latin presumably made by Adelard of Bath in the 12th century, edited and an-

notated by Giovanni Compano. It included over 400 figures (a chore presenting, at the time, technical difficulties). A digital copy can be viewed on the Library of Congress website (accessed 7/17/2023). You can also download a PDF copy from this website, and the following image is from this site.



Liendū eſt aūt q̄ p̄ter has animi ꝯceptiones: ſiue cōes ſcias mul/tas alias que numero ſunt incōp̄henſibiles p̄termiſit Euclides: quarū hec ē vna. ¶Si due quantitates equales ad quālibet tertiaz eiuſdem generis comparentur ſimul erunt ambe illa tertia aut eque maiores: aut eque minores: aut ſimul eq̄les. ¶Item alia Quanta eſt aliqua quantitas ad quālibet aliam eiuſdem generis tantam eſſe quālibet ter/tiam ad aliquam quartam eiuſdem generis in quantitatibus continuis: hoc vni/uerſaliter verum eſt ſiue antecedentes maiores fuerint conſequentibus ſiue mino res. magnitudo enim decreſcit in infinitum. in numeris autem non ſic: ſed ſi fuerit p̄imus ſubmultiplex ſecundi: erit quilibet tertius eque ſubmultiplex alicui⁹ quar/ti: quoniā numerus creſcit in infinitū: ſicut magnitudo in infinitū minuitur.

Propoſitio .i.

Triangulum equilaterum ſup̄a datam lineam rectam col locare.

¶Eſto data linea recta. a. b. volo ſuper ipſam triangulū equilaterū cōſtituere ſuper alterā eius extremitatē. ſ. in puncto. a. ponam pedē circini immobilem: ⁊ alterū pedem mobilem extendam vſq; ad. b. ⁊ deſcribam ſm quantitatē ipſius linee eatē per ſecundā petitionē circulū. c. b. d. f.

The bottom of page 8 of the first printed version of the *Elements*. The figure illustrates Proposition I of Book I, which gives the construction of an equilateral triangle (in fact, two such triangles are given in the figure).

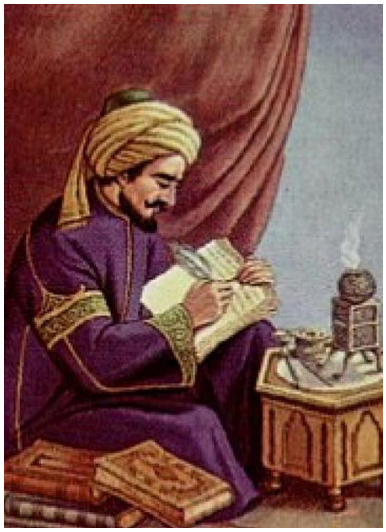
Note 5.3.F. As Europe fell into the "dark ages" (more appropriately, the "middle ages"; see Section 8.1. "The Dark Ages" (The Middle Ages)), the work of the classical period was preserved in the Arabic world. In the ninth century and afterward, many of the classical works of the ancient world were translated from Greek into Arabic. In the Introduction to his translation of the *Elements*, Heath lists 31 Arabic translators of the *Elements*. We now mention a few of these.

1. Abu'l Abbas al-Fadl (or Al-Nayrizi; circa 865–circa 922) wrote a commentary on Euclid's *Elements* which has survived. His commentary on books I through VI are in the Codex Leidensis, which can be viewed online at the Wilbour Hall

Mathematics and Mathematical Astronomy webpage (accessed 7/19/2023). His commentary on Books I through X were translated into Latin by Gherard of Cremona (or "Gerard"; 1114–1187) in the 11th century. We'll see more of Gherard of Cremona in Section 8.2. The Period of Transmission; see Note 8.2.C. This note is based on Heath's page 85 and Eves page 144.

2. Mohammad Abu'l-Wafa (June 10, 940–July 15, 998), one of the greatest Arabian mathematicians, wrote a commentary on the *Elements*, but did not complete it. He also wrote commentaries on Diophantus (see Section 6.8. Diophantus) and al-Khwarizmi (see Section 1.9. The Hindu-Arabic Numeral System, Note 1.9.I), but all of these are lost. This note is based on Heath's page 85 and the MacTutor biography webpage of Abu'l-Wafa.

Mohammad Abu'l-Wafa
(June 10, 940–July 15, 998)



Abu Al-Kindi
(circa 801–873)

Thabit ibn Qurra
(836–February 18, 901)

al-Hasan al-Haytham
(965–1040)

Nasir al-Din al-Tusi
(February 18, 1201–June 26, 1274)

Each image above is from the corresponding MacTutor biography webpage, except for the image of al-Kindi which is from traditionalarabicmusic.com (each accessed 7/18/2023).

3. Abu Al-Kindi (circa 801–873), is the author (1) of a work on the objects of Euclid's *Elements*, (2) of a book on the improvement of Euclid's *Elements*, and (3) of another work on Books XIV and XV of the *Elements* (though these are now known not to be by Euclid; see Note 5.3.A). His writings cover logic, philosophy, geometry, arithmetic, music, and astronomy. His geometrical works explore Archimedes' relationship between the diameter and circumference of a circle (as explained in Note 4.8.B of Section 4.8. A Chronology of π), and other results of Archimedes related to constructions and approximation. This note is based on Heath's page 86.

4. Thabit Qurra (836–February 18, 901) revised translations of Euclid's *Elements*, *Data*, and *On Divisions of Figures*. He is also reported to have written (1)

On the Premises of Euclid, (2) On the Propositions of Euclid, (3) On the propositions and questions which arise when two straight lines are cut by a third (which relates to a "proof" of the Parallel Postulate), and other works on geometry. He also translated parts of Archimedes, and Books V through VII of the *Conics* of Apollonius. This note is based on Heath's page 87.

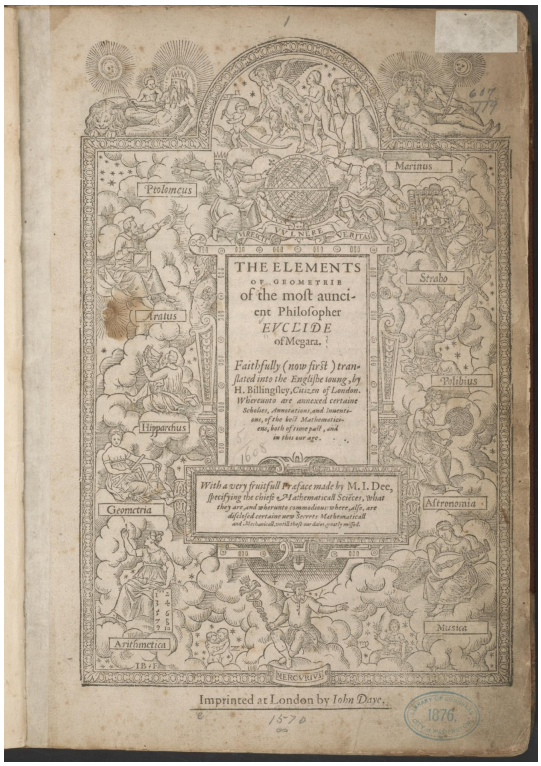
5. al-Hasan al-Haytham (965–1040) was a preeminent mathematician of his time. He wrote a large number of works on the *Elements* the titles of which include: (1) Commentary and abridgement of the *Elements*, (2) Collection of the Elements of Geometry and Arithmetic drawn from the treatises of Euclid and Apollonius, (3) Collection of the elements of the Calculus deduced from the principles laid down by Euclid in his *Elements*, (4) Treatise on measure after the manner of Euclid's *Elements*. He also wrote memoirs and commentaries on parts of the *Elements* in Books I, V, X (on the method of exhaustion), XII, and on the definitions used by Euclid. This note is based on Heath's pages 88 and 89.

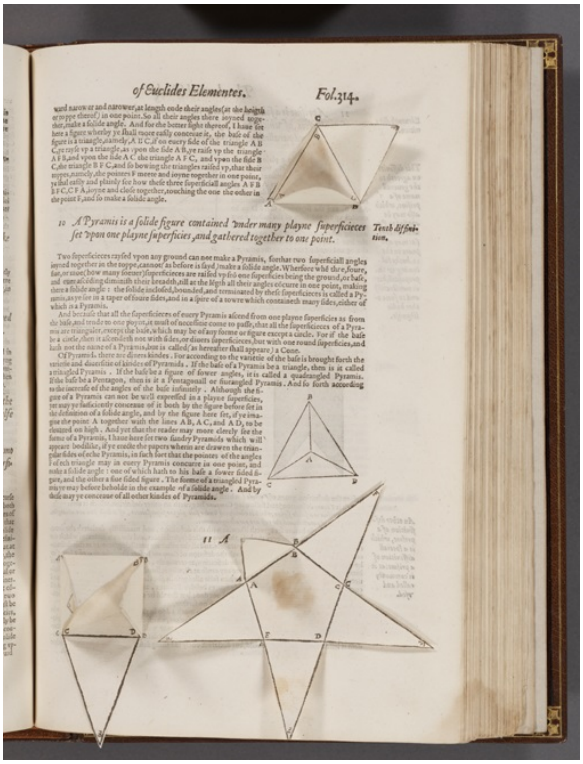
6. Nasir al-Din al-Tusi (February 18, 1201–June 26, 1274) wrote: (1) A treatise on the postulates of Euclid, (2) A treatise on the 5th postulate (this was possibly part of the first treated), (3) Principles of Geometry taken from Euclid, and (4) 105 problems out of the *Elements*. He also edited the *Data*. This note is based on Heath's page 89.

Note 5.3.G. The first English translation of the *Elements* appeared in 1570. It was translated by Sir Henry Billingsley (?–1606) and titled *Elements as The elements of*

geometrie of the most ancient philosopher Euclide of Megara. The title is in error, since "Euclid of Megara" (circa 435 BCE–circa 365 BCE) was a philosopher who has historically been often confused with "Euclid of Alexandria" (circa 325 BCE–circa 265 BCE). It was printed in London by John Daye, and includes a preface by English mathematician, geographer, and astrologer John Dee (he was astrologer to Queen Mary, but was imprisoned for being a magician; July 13, 1527–March 26, 1609). There is a MacTutor biography webpace of Dee (accessed 7/19/2023). The highly-illustrated title page of the book is given in the figure below (left), which is from the Library of Congress website for Billingsley's book (accessed 7/19/2023). There was some controversy that Billingsley did not write the translation, but that in fact it was written by Dee. This resolved in the late 19th century with the discovery of the copy of Theon's Euclid used by Billingsley, which contained marginal notes and comments Billingsley made while preparing his translation (it is now housed in the Princeton University Library). The work contains 928 pages (excluding the lengthy preface by Dee; the total length of the book was over 1000 pages). The notes include all the most important from the Greek commentaries of Proclus and the others, down to commentaries of Billingsley's time. It includes all 15 books of Euclid; since this is based on work of Theon, it mistakenly attributes Books XIV and XV to Euclid (see Note 5.3.A). Billingsley also includes a Book XVI. This was written by the French mathematician Franciscus Flussas Candalla (1512–1594) in 1566 and it compares the Platonic solids to one another (see Audrey Price's Ph.D. dissertation in history from the University of California, San Diego, titled *Pure and Applied: Christopher Clavius's Unifying Approach to Jesuit Mathematics Pedagogy*, page 31; this can be viewed online at the University of California's eScholarship website,

accessed 7/19/2023). Billingsley also include "pop up" models when addressing sold geometry. On Folio 314 (a "folio" is a sheet of paper; it was common at the time to number the folios and not the "pages" as we do today) there are three fold up paper models that give pyramids with (regular) triangular, square, and pentagonal bases. This is illustrate below in the figure on the right. The pop up pieces are not present in the Library of Congress version (see image 691 in that version). The image below is from the webpage for Frank Swetz and Victor Katz, "Mathematical Treasures—Billingsley Euclid," *Convergence* (January 2011).





Henry Billingsley attended Saint Jon's College, Cambridge in 1551 and is said to have studied at Oxford, but he did not earn a degree at either university. He was a wealthy merchant who was sheriff of London in 1584, and served as Lord Mayor in 1596. He founded scholarships for poor students at Saint John's College, and gave the college some tenement properties in 1591. This note is based on Heath's

pages 109 and 110, and the Mathematical Treasures—Billingsley Euclid webpage. All webpages used in this note were accessed 7/19/2023).

Note 5.3.H. Heath mentions 16 other English versions of the *Elements* (see his pages 110–112). We only discuss a few of these here. Isaac Barrow (October 1630–May 4, 1677), who we will see again in Section 11.8. Wallis and Barrow in connection with early developments of calculus, published an edition of the *Elements* in Latin in 1655. In 1660, he produced an English version which was published in London. It included all "15" books of the *Elements*, and new editions appeared in 1705, 1722, 1732, and 1751. Scottish mathematician Robert Simson (October 14, 1687–October 1, 1768) published both a Latin and an English edition of the *Elements* in 1756. The English edition went through some twenty-six editions, the last one appearing in 1844. Simson's version covered Books I through VI, XI and XII and was the standard text for many years. There were a total of 70 different editions, revision, and translations which appeared in Scotland, Ireland, England, Europe, and the United States.

This note is based on Heath's page 111, and the MacTutor biography webpage on Simson (accessed 7/19/2023), the source of the following image.

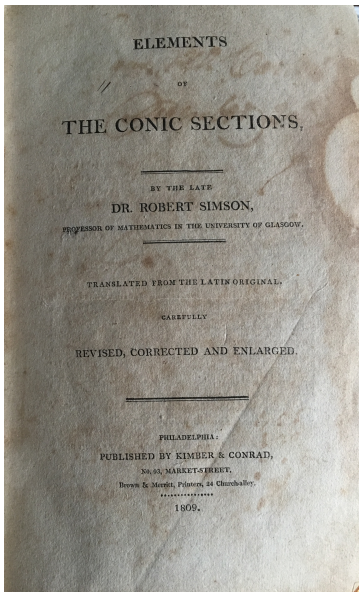
Note 5.3.I. John Playfair (March 10, 1748–July 20, 1819) published an edition of the *Elements* in 1795 which went into 10 editions over the next 50 years. His version was intended for use by students. Playfair used algebraic notation to abbreviate the proofs when he was teaching. One of the 18th century challenges of teaching geometry was dealing with the complicated nature of the Fifth Postulate (i.e., the Parallel Postulate; this is explained more in Section 5.7. Formal Aspect of the "Elements"). One can show that, assuming the Parallel Postulate of Euclid that: "Given a line an a point not on the line, it is possible to draw exactly one line through the given point parallel to the line." In fact, one can assume this result as a postulate (called Playfair's Axiom; an axiom and postulate are the same in modern terminology) and the postulates of Euclid *other than* the Parallel Postulate and then prove the Parallel Postulate. In other words, the Parallel Postulate and Playfair's Axiom are equivalent. So by replacing Euclid's (complicated) Fifth Postulate with (relatively simple) Playfair's Axiom, all of the same propositions of Euclid follow. In his book, Playfair stated his axiom in the equivalent form: "Two straight lines cannot be drawn through the same point, parallel to the same straight line, without coinciding with one another." This has become widely known as Playfair's Axiom (or, if Euclid's Fifth Postulate is assumed, then Playfair's Theorem). However, it had already been given in the fifth century by Proclus (circa 411–April 17, 483) and Playfair even points this out. This note is based on Heath's pages 111 and 112, and the MacTutor biography webpage on Playfair (accessed 7/19/2023), the

source of the below image.

Playfair's Axiom is easily negated when in the form "Given a line an a point not on the line, it is possible to draw exactly one line through the given point parallel to the line." We would negate the "exactly one line" part, which gives us two options: (1) no lines, and (2) more than one line. One can show that if there are more than one such line, then there are infinitely many such lines. Negating Playfair's Axiom would be equivalent to negating Euclid's Parallel Postulate. Such a negation results in *non-Euclidean geometry*. In the first case of no lines as described, we get the form of non-Euclidean geometry called *elliptic geometry*. The second case gives *hyperbolic geometry*. These ideas are explained in more detail in Section 13.8. Non-Euclidean Geometry, Supplement. Hyperbolic Geometry, and Supplement. A Quick Introduction to Non-Euclidean Geometry.

Note. As a passing comment, we observe that because of the large number of editions of some of the early English versions of the *Elements*, it is possible to find old copies for sale at moderate prices. I have collected a handful of such books,

paying $10 to $25 each for copies in rough condition (detached or missing covers, some water damage, etc.) on E-Bay. I have an 1809 edition of Robert Simson's *Elements of The Conic Sections*, an 1825 edition of Simson's *The Elements of Euclid*, and an 1842 edition of John Playfair's *Elements of Geometry*. The title pages of each are given below.



ELEMENTS

OF

THE CONIC SECTIONS,

BY THE LATE

DR. ROBERT SIMSON,

PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF GLASGOW.

TRANSLATED FROM THE LATIN ORIGINAL,

CAREFULLY

REVISED, CORRECTED AND ENLARGED.

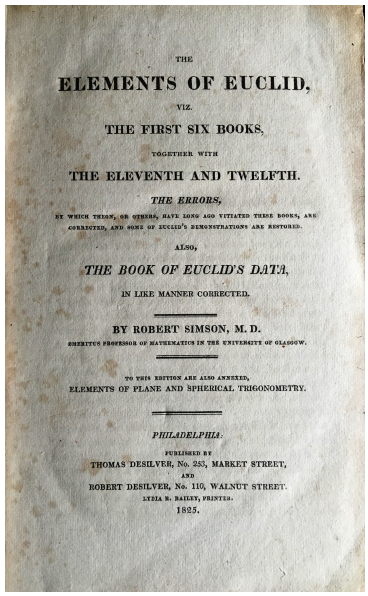
PHILADELPHIA:

PUBLISHED BY KIMBER & CONRAD,

No. 93, MARKET-STREET,

Brown & Merrett, Printers, 44 Churchalley.

1809.



THE

ELEMENTS OF EUCLID,

VIZ.

THE FIRST SIX BOOKS,

TOGETHER WITH

THE ELEVENTH AND TWELFTH.

THE ERRORS,

BY WHICH THESE, OR OTHERS, HAVE LONG AGO VITIATED THESE BOOKS, ARE CORRECTED, AND SOME OF EUCLID'S DEMONSTRATIONS ARE RESTORED.

ALSO,

THE BOOK OF EUCLID'S DATA,

IN LIKE MANNER CORRECTED.

BY ROBERT SIMSON, M. D.

EMERITUS PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF GLASGOW.

TO THIS EDITION ARE ALSO ANNEXED,

ELEMENTS OF PLANE AND SPHERICAL TRIGONOMETRY.

PHILADELPHIA:

PUBLISHED BY

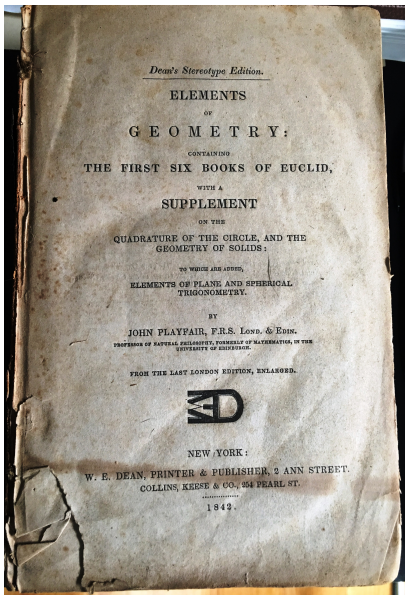
THOMAS DESILVER, No. 253, MARKET STREET,

AND

ROBERT DESILVER, No. 110, WALNUT STREET.

LYDIA R. BAILEY, PRINTER.

1825.



Dean's Stereotype Edition.

ELEMENTS

OF

GEOMETRY:

CONTAINING

THE FIRST SIX BOOKS OF EUCLID,

WITH A

SUPPLEMENT

ON THE

QUADRATURE OF THE CIRCLE, AND THE GEOMETRY OF SOLIDS:

TO WHICH ARE ADDED,

ELEMENTS OF PLANE AND SPHERICAL TRIGONOMETRY.

BY

JOHN PLAYFAIR, F.R.S. Lond. & Edin.

PROFESSOR OF NATURAL PHILOSOPHY, FORMERLY OF MATHEMATICS, IN THE UNIVERSITY OF EDINBURGH.

FROM THE LAST LONDON EDITION, ENLARGED.

NEW YORK:

W. E. DEAN, PRINTER & PUBLISHER, 2 ANN STREET.

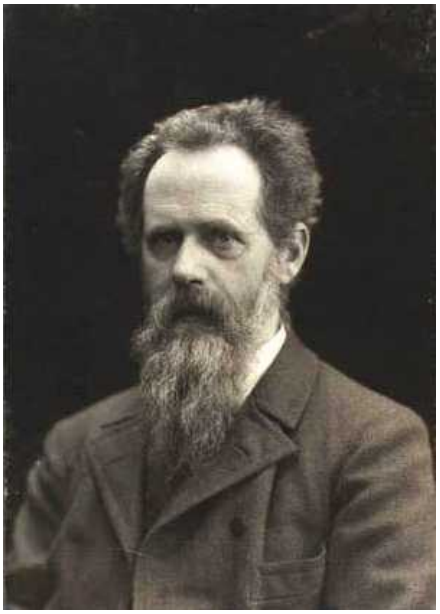
COLLINS, KEESE & CO., 254 PEARL ST.

1842.

Note 5.3.J. Johan Ludvig Heiberg (November 27, 1854–January 4, 1928) was a Danish (i.e., from Denmark) philologist (a linguist who studies historical sources). Johan Heiberg attended Aalborg Cathedral School and the University of Copenhagen, earning a doctorate in philology in 1879. He was a professor of Philology at the University of Copenhagen from 1896 to 1924 (and also as a professor of archeology from 1896 to 1911). In all, he produced over 200 publications. He was a member of the Royal Danish Academy of Sciences and Letters starting in 1893 and served as the editor of its journal form 1902 to 1913. He published a two volume version of Apollonius' *Conics* (in classical Greek) in 1891 and 1893; these can be viewed

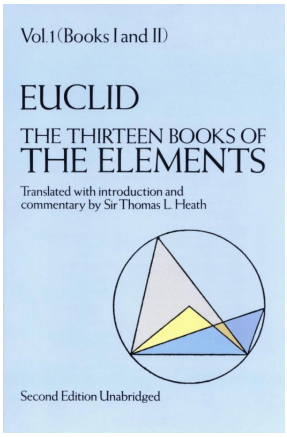
online (in PDF) at the Wilbour Hall Mathematics and Mathematical Astronomy webpage; accessed 7/21/2023). He published an edition of Ptolemy's *Almagest* in classical Greek in 1898; this can be viewed online (in PDF) at the Wilbour Hall Mathematics and Mathematical Astronomy webpage; accessed 7/21/2023). Heiberg studied the famous "Archimedes Palimpsest" in Constantinople in 1906. He was allowed to photograph it and he produced transcriptions which he published in a "complete works" of Archimedes (in Latin and Greek) between 1910 and 1915. For more on this, see Section 6.2. Archimedes and the online PowerPoint presentation of "Supplement. Archimedes: 2,000 Year Ahead of His Time". He also published work on the Greek mathematicians Serenius of Antinoupolis (1896) and Heron of Alexandria (in 1899). Of importance to this section is the fact that during the 1880s, Heiberg (along with Heinrich Menge) published a several volume work, titled *Euclidis Opera Omnia*. It is written in Latin and Greek. The first three volumes cover Euclid's *Elements* (or "*Euclidis Elementa*"). Volume 1 covers Books I through IV (1883) and is online on the Archive.org, Volume 2 covers Books V through IX (1884) and it online on the Archive.org. Richard J. Trudeau in his *The Non-Euclidean Revolution* (Birkhauser, 1987) states (see his page 23): "...a new Greek text was compiled in the 1880s by the Danish philologist Johan L. Heiberg, which is probably the closest scholars will ever come to reconstructing the original." His primary two sources in this work were the Vatican Manuscript Number 190 (see Note 5.3.A) and the Bodleian Manuscript (see Note 5.3.E). The Greek version of Hieberg is in print today with a new accompanying English translation as Richard Fitzpatrick's *Euclid's Elements of Geometry*, Revised and corrected (2008). This note (and the image below) is based on the Wikipedia webpages on Heiberg and

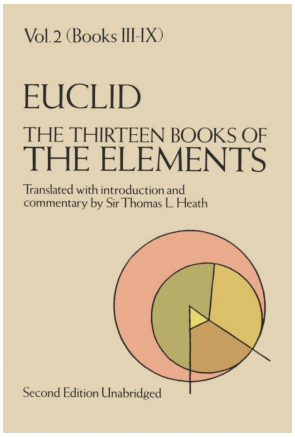
the Archimedes palimpsest (accessed 7/21/2023)

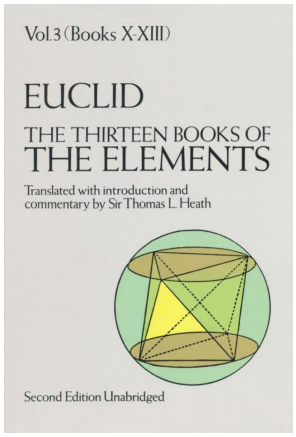


Note 5.3.K. Thomas Little Heath (October 5, 1861–March 16, 1940) was a British civil servant who was active in translating classical works of Greek mathematics into English. He attended Trinity College starting in 1879 and graduates with a degree in both mathematics and classics. He took the civil service exam in 1884 and then worked for the Treasury, becoming the permanent secretary to the Treasury in 1913. He moved over to the National Debt Office in 1919 where he worked until he retired in 1926. Even though he held down these full-time jobs, he was still able to become a leading expert in the history of mathematics. He was elected as a Fellow of the Royal Society in 1912 and served as president of the Mathematical Association (established to improve the teaching of geometry in England), and a fellow of the British Academy. While still an undergraduate, he wrote articles for the *Encyclopeadia Britannica* on 'Pappus' and 'Porisms.' In 1885, he published his first book, *Diophantus of Alexandria: A Study in the History of Greek Algebra.*

This can be viewed on Archive.org (the second edition published in 1910 is also on Archive.org); we'll refer to this source again in Section 6.8. Diophantus. In 1896 he published *Apollonius of Perga: A Treatise on Conic Sections* (using modern notation), which included a preface on previous Greek work on conic sections. This can be viewed on Archive.org; we'll refer to this source again in Section 6.4. Apollonius. *The Works of Archimedes* was published in 1897. With Heiberg's discover of Archimedes *The Method* in the "Archimedes Palimpsest' (see Note 5.3.J), Heath added a supplement to his *The Works of Archimedes* in 1912, *The Method of Archimedes, Recently Discovered. A Supplement to The Works of Archimedes.* The 1912 version can be viewed on Archive.org; we'll refer to this source again in Section 6.2. Archimedes.

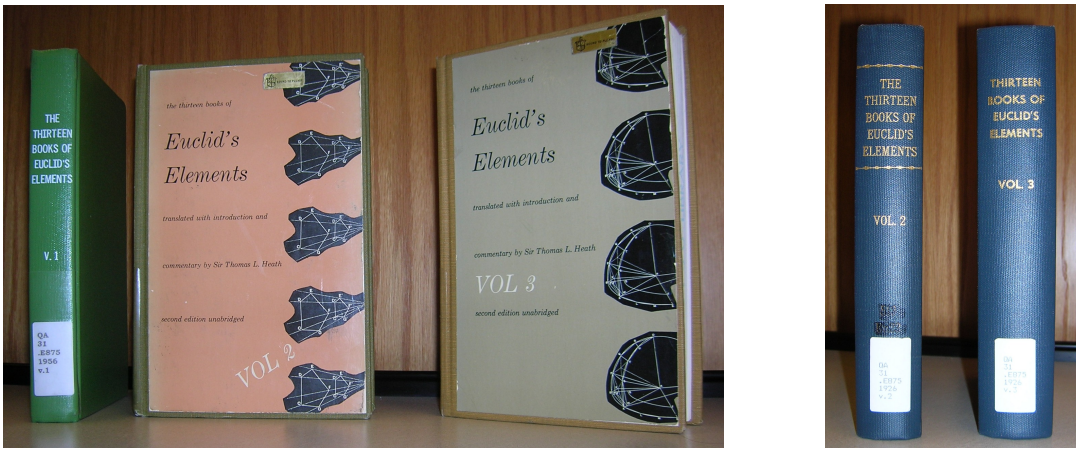






The reason we call attention to Thomas Heath is that he produced the standard English translation of Euclid's *Elements.* In 1908 he published a three volume translation of all of the books of the *Elements* and his main source was Heiberg's translation (see Note 5.3.J), as is revealed by the full title: *The Thirteen Books of Euclid's Elements, Translated from the Text of Heiberg, with Introduction and Commentary.* Volume 1 covers Books I and II, Volume 2 covers Books III through IX, and Volume 3 covers Books X through XIII. A second edition ("Revised with

Additions") was published in 1926. This work (as well as most of the other works of Heath which we mention in this note) was published by Cambridge University Press. In fact, the ETSU Sherrod Library has copies of Volumes 2 and 3 of the 1926 second edition (LOC numbers QA31.E875 1926 v.2 and QA31.E875 1926 v.3); Volume 1 is likely lost. In 1956, Dover Publications reprinted the three volumes of the 1926 edition in inexpensive paperback form. The Sherrod Library also has a copy of this (all three volumes; the paperback covers have been replaced with new hardback covers).

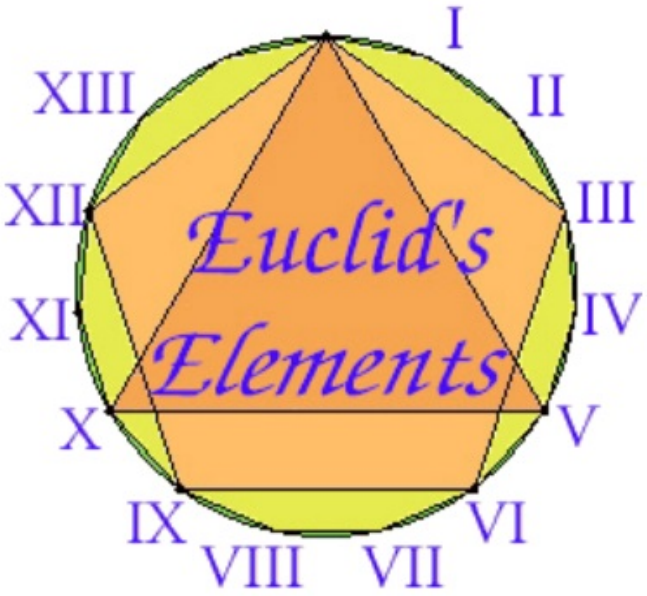


The copies of Heath's translations of the *Elements*

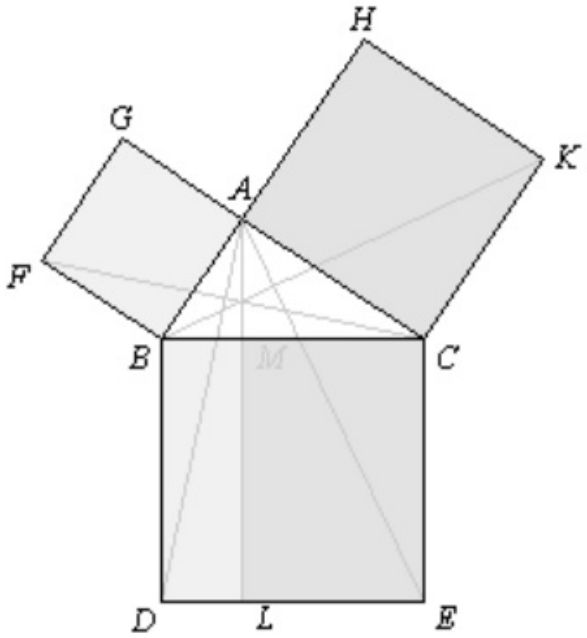
The images of the three volumes given at the beginning of this note are the versions of the Dover editions currently available. By the way, Dover Publications has *many* classical books in print on almost every topic! In fact, they have editions of other books by Heath, including *The Works of Archimedes* (originally: Cambridge University Press, 1897 and 1912), *Aristarchus of Samos: The Ancient Copernicus* (Clarendon Press, 1913), *A History of Greek Mathematics, Volume 1: From Thales to Euclid* and *A History of Greek Mathematics, Volume 2: From Aristarchus to Diophantus* (Clarendon Press, Oxford, 1921), *A Manual of Greek Mathematics* (Oxford University Press, 1931; this is a condensed version of the two volume *A*

History of Greek Mathematics), and *Greek Astronomy* (J.M. Dent & Sons, 1932). These are in the public domain, and you can probably find them several places online. In particular, Heath's three volumes of the *Elements* are on Archive.org: Volume 1, Volume 2, and Volume 3.

Note 5.3.L. As we've seen in this section there are numerous copies of the *Elements* online, including those of utmost historical importance. Probably the best online version of the *Elements* is one that was written in the 1990s. It is by David Joyce of Clark University in Worcester, MA (click on the logo to go to the website):



It includes diagrams, but they are static. Another online version of the first four books of the *Elements* is by J. T. Poole of Furman University in Greenville, SC. It includes dynamic figures that change as your progress through a proof. It was created in 2002. Click on the logo to go to the website:



Euclid
The Elements, Books I - IV

Revised: 8/14/2023