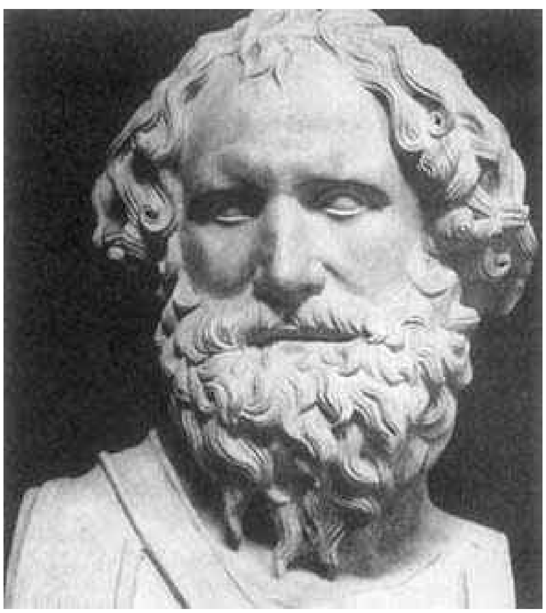
6.2. Archimedes

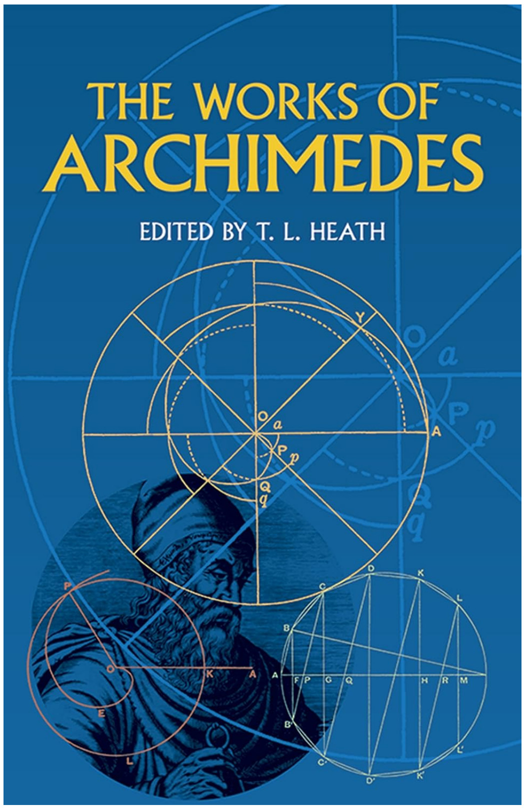
Note 6.2.A. In E. T. Bell's *Men of Mathematics* (Simon and Schuster, 1937) he states the following on his page 20: "Any list of the three 'greatest' mathematicians of all history would include the name of Archimedes. The other two usually associated with him are Newton (1642–1727) and Gauss (1777–1855)." (We first mentioned Bell's book in Section 3.2. Pythagoras and the Pythagoreans; see Note 3.2.A for a somewhat critical view.) Eves says (on his page 165): "One of the greatest mathematicians of all time, and certainly the greatest of antiquity, was Archimedes. . . ." He was killed during the sacking of Syracuse by the Romans in 212 BCE at the age of 75, so that his life spanned from 287 BCE to 212 BCE.

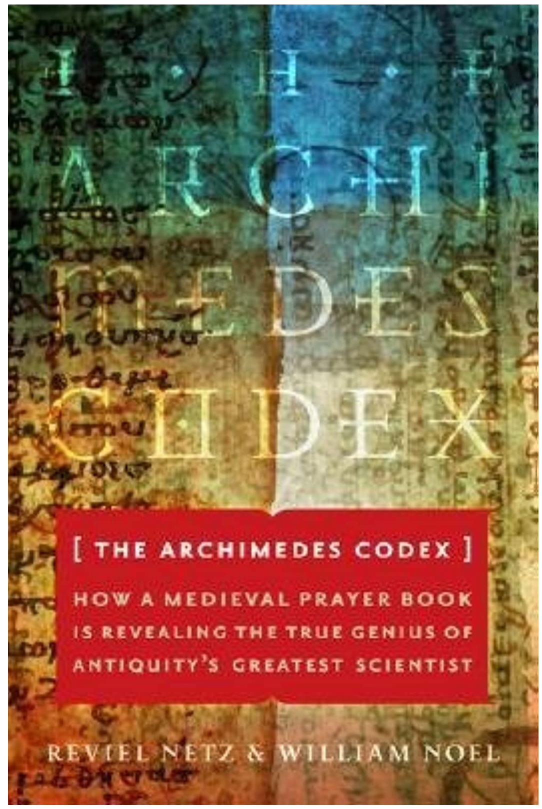


Images from the MacTutor biography webpage on Archimedes (left) and Worthpoint.com (right); both accessed 9/22/2023

In addition to Eves, the sources for our notes on Archimedes are Chapter XIII of Thomas Heath's *A History of Greek Mathematics, Volume 2* (Oxford: Clarendon Press, 1921), Thomas Heath's *The Works of Archimedes* (Cambridge University

Press, 1897) (both of Heath's books are available from Dover Publications), and Reviel Netz and William Noel *The Archimedes Codex: How a Medieval Prayer Book is Revealing the True Genius of Antiquity's Greatest Scientist* (Da Capo Press, 2007).





Heath's book has a 172 page introduction before giving translations of *On the Sphere and Cylinder*, *Measurement of a Circle*, *On Conoids and Spheroids*, *On Spirals*, *On the Equilibrium of Planes*, *The Sand Reckoner*, *Quadrature of the Parabola*, *On Floating Bodies*, *Book of Lemmas*, *The Cattle Problem*, and the *Method*. Heath's 1897 edition of *The Works of Archimedes* does not include the *Method* since it was not rediscovered until 1906. The version of this in print today by Dover publications does include the *Method*, though it is work of Heath's from 1912. It is commented in the *The Archimedes Codex* that (see pages 30 and 31):

> "For Archimedes [in English] there existed only T. L. Heath's poor paraphrase published in 1897, which simply ignores Archimedes' mathematical language. ... If you open an edition from that era, the diagrams you find are not based upon what is actually drawn in the original manuscripts. The diagrams represent, instead, the editor's own drawing. I [Will Noel] was shocked to realize this and began to consider whether I should produce, for the first time, an edition of the diagrams."

We'll discuss Noel's work in Supplement. Archimedes' *Method*. We should mention that the year of birth of Archimedes given above (287 BCE) is based on, as stated on page 33 of *The Archimedes Codex*, the writings of Johannes Tzetzes (circa 1110–1180) in the 12th century. Tzetzes is also the main source of the story that Archimedes designed mirrors that could be used to burn ships (*The Archimedes Codex* declares this as "just that—a story," page 33).

Note 6.2.B. Archimedes of Syracuse (287 BCE–212 BCE) was the son of Phidias the astronomer. The ancient Greek historian Diodorus (circa first century BCE) wrote that Archimedes spent time in Egypt, where he invented the Archimedean screw for pumping water. In Egypt, it is likely that the studied in Alexandria with the successors of Euclid (he likely met Eratosthenes there, to whom he sent a copy of his *Method* and an introductory letter that has survived). Tradition has it that Archimedes designed machines to defend Syracuse during the Roman siege. He is said to have designed catapults that could function at different ranges, long poles that could protrude out of the defensive walls of the city over the coast to either drop heavy weights on attacking enemy ships, and grappling hooks that could grab onto ships and be used to hoist the ships up in the air to be dropped and destroyed. The existence, use, and reported effectiveness of these machines may be apocryphal, but there is very solid evidence of Archimedes' abilities that survive in copies of his works and commentaries refereeing to his works. This note is based on Heath's *A History of Greek Mathematics, Volume 2*, pages 16–17.

Note 6.2.C. Another story is related to Archimedes and a goldsmith who built a crown for King Hieron (or Hiero) II of Syracuse (circa 308 BCE–215 BCE) from a given weight of gold. There was concern that the goldsmith replaced some of the gold with silver. The question of how to test this without destroying the crown was put to Archimedes. The legend goes that Archimedes discovered the solution to the problem (in the form of the First Law of Hydrostatics) when taking a public bath and realizing that a body in water is buoyed be a force equal to the weight of the displaced water. This would allow him to find the volume of the crown by submerging it in water, weighing the displaced water, and using the density of water to find the corresponding volume. The density of gold can then be used to see if the crown has the proper weight to be of its known volume (the crown was found to weigh less than it should, revealing the use of something less dense than gold). The legend says that Archimedes in his excitement over his discovery jumped out of the bath and ran down the street naked shouting "Eureka" (meaning "I have found it"). This story is relayed to us through the Roman architect and engineer Vitruvius (circa 75 BCE–circa 15 BCE), who wrote over 200 years after Archimedes lived. It is plausible that the story is meant as a popular way to discuss Archimedes' work

Floating Bodies, though *The Archimedes Codex* mentions that the key observation (which in the book is summarized as “bigger things make bigger splashed,” page 34) is so trivial as not be be mentioned in *Floating Bodies*. Concerning this story and others, Netz and Noel go on to comment (see *The Archimedes Codex* pages 34 and 35):

> “Vitruvius clearly knew nothing about Archimedes’ science. This is the pattern with all of the stories dealing with Archimedes, from Vitruvius to Tzetzes. They appear to be urban legends. Sorry.”

Note 6.2.D. Greek philosopher and historian Plutarch (circa 46 CE–circa 119 CE) gives three versions of Archimedes’ death (according to the MacTutor biography webpage on Archimedes; accessed 9/24/2023). Each involves Archimedes being killed by one or more Roman soldiers. Plutarch’s version given in *The Archimedes Codex* (see page 62) is:

> “He was by himself, working out some problem with the aid of a diagram. . . he was not aware of the incursion of the Romans or of the capture of the city. Suddenly a soldier cam upon him and ordered him to go with him to Marcellus [circa 268 BCE– 208 BCE; the Roman general who captured Syracuse during the Second Punic War]. This Archimedes refused to do until he had worked out his problem and established its demonstration, whereupon the soldier flew into a passion, drew his sword, and killed him.”

See the figure below (left).


The Death of Archimedes by Thomas Degeorge (1815) from the Wikipedia page on Archimedes (left), and CityMap website of Sicily (right; both webpages accessed 9/24/2023).

Marcellus had developed respect for Archimedes due to the defensive weapons he had invented and which were used against the Romans. He gave orders that no harm should be done to Archimedes. Upon learning of Archimedes' death, Marcellus had him buried in in the city cemetery. Archimedes had requested that his tombstone have an engraving of a sphere circumscribed by a right cylinder (he showed in *On the Sphere and Cylinder* that volume of the sphere is $\frac{4}{3}\pi r^3$ and the volume of the cylinder is $2\pi r^3$ in a corollary to Proposition 34; see Note AW.D in Supplement. The Content of Archimedes' Work, Part 1). Marcellus saw to it that this was carried out. Years later in 75 BCE the Roman statesman Cicero (January 3, 106 BCE–December 7, 43 BCE) was serving in Sicily and set out to find Archimedes' tomb. The locals knew nothing about it, but he was able to track down the unique tombstone with the inscription in an overgrown part of the cemetery. He had the area cleared and ordered that it be preserved. With time, it was lost again. It remains lost today (even though Eves says that it was found in 1965). In the Park of Neapolis in Syracuse, is located the Necropolis Grotticelle.

According to local stories this includes "Archimedes' Grave," though it is actually a Roman catacomb which dates centuries after the death of Archimedes; it is best described as a "tourist trap," though if you Google "Archimedes' tomb" it may turn up the lcoation. See the figure above (right). The source for this note is Eves' page 166.

Note 6.2.E. Eves opens his discussion of Archimedes' mathematics with (see page 167):

> "The works of Archimedes are masterpieces of mathematical exposition and, to a remarkable extent, resemble modern journal articles. They are written with a high finish and an economy of presentation and exhibit great originality, computational skill, and a rigor in demonstration."

The extant works of Archimedes, according to Heath's *A History of Greek Mathematics, Volume 2* (1921; see page 22), are: *On the Sphere and Cylinder* (two books), *Measurement of a Circle*, *On Conoids and Spheroids*, *On Spirals*, *On the Equilibrium of Planes* (two books), *The Sand Reckoner*, *Quadrature of the Parabola*, *On Floating Bodies* (two books), *Stomachion* (a fragment), and *The Method*. Oddly, this does not mention *Book of Lemmas* and *The Cattle Problem*, which are included in Heath's *The Works of Archimedes* (1897). Archimedes geometric work mainly consists of finding areas of curved plane figures and and volumes of curved surfaces. These works pick up where Euclid's Book XII of the *Elements* left off (recall that Euclid shows the area of a circle is πr^2 in Proposition XII.2, and the volume of a right circular cone is $\frac{1}{3}\pi r^2 h$ in Proposition XII.10; see Note 5.4.P of Section 5.4. Content of the "Elements").

Note 6.2.F. He found areas of a parabolic segment (in *Quadrature of the Parabola*) and a spiral (in *On Spirals*), the surface area and volume of a sphere (in *On the Sphere and Cylinder*), and volumes generated by segments of solids of revolution of degree two. Notice that these are problems today solved with integration in Calculus 1 and 2 (MATH 1910 and 1920). Integration began with Archimedes and reached its mathematical completion (minus some rigorous polishing) with Newton and Leibniz between the mid 1660s and the mid 1680s. In arithmetic, as we saw in Section 4.8. A Chronology of π (see Note 4.8.B), and in more detail in Archimedes: 2,000 Years Ahead of His Time (in PowerPoint) and the transcript of the presentation (in PDF), that he gave an accurate approximation of π using polygons in his *Measurement of a Circle*. In the process of his computation, he gave approximations to the value of square roots of several non-square numbers (these details are given in the PowerPoint presentation just mentioned). In *On Floating Bodies* he invented the science of hydrostatics. *On the Equilibrium of Planes* rigorously establishes the centers of gravity (or "centroids") of certain two-dimensional regions. In *On Conoids and Spheroids*, Archimedes considers several volume problems, including the volumes of paraboloids of revolution and hyperboloids of revolution (these are the "conoids"), and revolutions of an ellipse about its major axis and its minors axis (these are the "spheroids"). In *The Sand Reckoner*, Archimedes gives a system for expressing very large numbers which were beyond the Greek arithmetical notation of the time. His system could be used to express number up to the size, in our modern notation, that would require 80,000 million million digits (i.e., 8×10^{16} digits). Of particular historical note is a statement in *The Sand Reckoner* that Aristarchus (circa 310 BCE–circa 230 BCE) advocated a heliocentric ("sun centered") model of the universe; this is a theory often attributed to Copernicus (February 19, 1473–May 24, 1543). Much of the content of *The Method* is discussed in Supplement. Archimedes' Method, Part 1 and Supplement. Archimedes' Method, Part 2. A more detailed discussion of Archimedes' body of work is given in Supplement. The Content of Archimedes' Work, Part 1 and Supplement. The Content of Archimedes' Work, Part 2.

Revised: 4/29/2024