6.7. Ancient Greek Algebra

Note. This section marks a transition from the geometric work of recent sections into the work of Diophantus (circa 200–circa 284), who is covered in the next section, and whose work includes algebraic topics (or more precisely, number theory). We discuss three "stages" of the evolution of algebra, and problems explored in the *Greek Anthology*, which dates from around 500 CE. The notes of this section are largely based on Heath's *A History of Greek Mathematics* Volume 2 pages 441–443 and 455–456.

Note 6.7.A. Georg H.F. Nesselmann (February 14, 1811–January 7, 1881) was a linguist and a mathematics historian. In 1842, he published *Versuch einer kritischen Geschichte der Algebra* [Attempt at a Critical History of Algebra], (Berlin: Verlag von G. Reimer). A copy is available (in German) on Google Books (accessed 5/16/2024). In this, he gave three stages in the evolution of algebra: (1) Rhetorical Algebra, (2) Syncopated Algebra, and (3) Symbolic Algebra. The Rhetorical Algebra stage involves a complete absence of any symbols, and problems are stated and solved entirely in prose (i.e., with a verbal-only presentation). *Heath, Volume 2* page 455 states (presumably following Nesselmann's commentary): "The first stage is represented by such writers as Iamblichus, all Arabian and Persian algebraists, and the oldest Italian algebraists and their followers, including Regiomontanus." Iamblicus (circa 250–circa 325) was introduced in Section 3.3. Pythagorean Arithmetic; see Note 3.3.A. Regiomontanus (also known as Johann Múller, June 6, 1436–July 6, 1476) will play a prominent role in Section 8.5. The Fifteenth Century. See

Note 8.5.D and also Note 8.5.C, where he is mentioned in connection to one of his teachers, Georg von Peurbach (May 20, 1423–April 8, 1461). Regiomontanus also plays a role in the history of trigonometry, as explained in my online notes for the history component of Introduction to Modern Geometry (MATH 4157/5157) on Section 5.2. Regiomontanus and Euler's Trigonometric Functions. The Syncopated Algebra stage marks the beginnings of symbolic manipulation by introducing the use of certain abbreviated symbols for recurring quantities and operations. *Heath, Volume 2* page 456 states: "To this stage being Diophantus and, after him, all the later Europeans until about the middle of the seventeenth century (with the exception of Vieta...)." Diophantus (circa 200–circa 284) is the topic of the next section, Section 6.8. Diophantus, and contributed much to number theory. François Viète (1540–December 13, 1603) is the topic of Section 8.9. François Viète and he was the first to use a systematic algebraic notation, introduced in his *Analytic Art* in 1591. The Symbolic Algebra stage is described in *Heath, Volume 2* page 456 as: "...[using] a complete system of notation by signs having no visible connexion with the words or things which they represent, a complete language of symbols, which entirely supplants the 'rhetorical' system, it begin possible to work out a solution without using a single word of ordinary language with the exception of a connecting word or two here and there used for clearness' sake." [As an editorial comment, your instructor would like to emphasize the desirability of using *lots of words* in presenting an algebraic argument in the classroom/textbook setting, not just "a connecting word or two." The plan in this setting, and MANY others, is to **clearly communicate** the argument, and a nearly-prose-free sequence of equations is not the best way to accomplish this!] Eves states (page 379): "Symbolic algebra made

its first appearance in Western Europe in the sixteenth century [with Viète], but did not become prevalent until the middle of the seventeenth century. It is not often realized that much of the symbolism of our elementary algebra textbooks is less than 400 years old."

Note 6.7.B. The *Greek Anthology* (also known as the *Palatine*) dates from around 500 CE and was compiled by Metrodorus, a Greek grammarian and mathematician who lived in the 6th century. There are 46 problems stated as concise, sometimes witty stories (i.e., "epigrams"). The problems lead to simple equations that you might encounter in a high school algebra class. Several of the problems involve dividing a number of objects among several people. An example of such a problem is given in Eves:

> **6.13(a). Problems from the "Greek Anthology"**
>
> How many apples are needed if 4 persons out of 6 receive $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{5}$, respectively, of the total number, while the fifth receives 10 apples, and 1 apple remains for the sixth person?

Plato (circa 427 BCE–348 BCE) in his *Laws* mentions such problems, as well as mixture problems. Plato's allusions to these types of problems show that they date back to at least the 5th century BCE. The problems in *Greek Anthology* are categorized by Heath (in *History, Volume 2*, pages 442 and 443). Twenty-three of the problems result in a simple equation in one unknown. Twelve problems result in simultaneous equations in two unknowns. One problem gives simultaneous equations in three unknowns, and another problem gives four equations in four unknowns (this is given by Eves in Problem Study 6.14(c)). Six of the problems

involve pipes or spouts that fill or empty a vessel; Eves' Problem Study 6.14(b) is of this sort. There are two indeterminant problems of the first degree which have an infinite number of integer solutions. One of these is given in Eves' Problem Study 6.13(c).

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