Chapter 8. European Mathematics, 500 to 1600

8.1. "The Dark Ages" (The Middle Ages)

Note 8.1.A. We put Eves' title of this section in quotation marks. A more modern term for "the Dark Ages" is "the Middle Ages" (also called the *medieval period*), which ranged from the fall of Rome in 476 CE to the beginning of the Renaissance (more vaguely defined, as sometime in the 15th century). The "Dark Ages" idea is due to Italian scholar Francesco Petrarch (July 20, 1304–July 18/19, 1374) who introduced it in the 1330s to contrast the "light" of classical antiquity with the "dark" of post-Roman Europe. Part of the impression of "darkness" is the scarcity of records for this time period. In the 19th and 20th century, as knowledge grew about the accomplishments of the time, the term "Dark Ages" was restricted to the 5th century through the 10th century (today, called the "Early Middle Ages"). Today, scholars largely avoid the term "Dark Ages" due to its negative association and implication of ignorance. This historical information is from the Wikipedia page on the Dark Ages (historigraphy) (accessed 6/18/2023). In fact, Seb Falk recently published *The Light Ages: The Surprising Story of Medieval Science* (W. W. Norton & Company, 2020) in which he argues that "Medieval people sought to build understanding of why things in nature behave as they do and used their understanding to make future predictions" (Falk's page 6) and "As medieval thinkers sought to understand the world around them, from the passing of the seasons to the stars in the sky, they came to develop a vibrant scientific culture" (from the

front flap). He also observes that the medieval studies would not be classified as "science" in the modern sense of the term and that they did not follow the "scientific method." After all, the scientific method is largely the product of the late 16th or early 17th century (though it has many predecessors), with Galileo (February 15, 1564–January 8, 1642) as an early pioneer.

Note. Eves states (page 258): "Schooling became almost nonexistent, Greek learning all but disappeared, and many arts and crafts bequeathed by the ancient world were forgotten. Only monks of the Catholic monasteries and a few cultured laymen preserved a slender thread of Greek and Latin learning." As we'll see, this is somewhat the case in Europe, but in other parts of the world (in particular the Arabic part of the world, including the southern shore of the Mediterranean) progress continued and appreciation for the accomplishments of classical Greece remained. Eves' description is of "the West during the whole of the half-millennium covered by the Dark Ages," so he seems to imply that his comments are aimed at Europe from about 500 CE to 1000 CE (i.e., the Early Middle Ages). Eves mentions four "Dark Age" European mathematicians, Boethius, Bede, Alcuin, and Gerbert, so we consider them in this section.

Note 8.1.B. Anicius Boethius (circa 480 CE–524 CE) was a Roman mathematician and philosopher who wrote texts on geometry and arithmetic that were used for instruction in monastic schools for centuries after his death. His work reveals that he was well-educated and fluent in Greek. His understanding of mathematics was

"rather limited." His *Arithmetic* was based on Nicomachus of Gerasa's (circa 60 CE–circa 120 sc ce) *Introductio Arithmeticae* or *De institutione arithmetica* and, though of "poor quality," *Arithmetic* was the best such text available and it was used for many centuries and used to teach the medieval scholars Pythagorean number theory. According to The British Library website on *De institutione arithmetica*:

> "Rather than a practical manual of calculation, it comprises a philosophical discussion of numbers, their relationships and meanings. One of the texts most influential features was its division of the mathematical sciences into arithmetic, music, geometry, and astronomy, which it together designated as the quadrivium. ... This manuscript copy of *De institutione arithmetica* was made in England in the third quarter of the 12th century. The text is illustrated with mathematical diagrams, including a multiplication table for Roman numerals..."

He is also known to have written a geometry text, but it does not survive. It consisted of statements of propositions from Euclid's Book I, and a few from Books III and IV (Eves, page 258). Most of Boethius' work is on philosophy. He translated and wrote commentaries on the works of Plato and Aristotle (though he died before completing has translation of Plato). Up to the 12th century his writings on logic, collectively called *Logica netus* ("The Old Logic"), were the main such works in Europe. Boethius' best known work is his *De consolatione philosophiae* ("Consolation of Philosophy"). It is written as a dialogue between Boethius and a woman who represents the spirit of philosophy. He considers the philosophy of Plato and the work is described as "written by a Platonist who is also a Christian." This work is in the public domain and you can read it on the Project Gutenberg page on *The Consolation of Philosophy* (accessed 6/19/2023). Boethius became

involved in arguments between the Church of Rome and the Eastern Orthodox Church in Constantinople (modern-day Istanbul, Turkey). Ultimately, Boethius was charged with treason, the practice of magic, and sacrilege. He wrote *De consolatione philosophiae* while in prison for these charges. He was executed for his "crimes" in 524. This history and the following image are from the MacTutor biography page of Boethius (accessed 6/18/2023).

Note 8.1.C. Bede (circa 673–May 26, 735) was an English monk, perhaps better-known as "The Venerable Bede." He was born in northeastern England, and served at the monestaries of Saint Peter and Saint Paul in the Kingdom of Northumbria (in modern-day Northern England and Southern Scotland). Eves states (page 259): "His numerous writings include some on mathematical subjects, chief of which are his treatises on the calendar and on finger reckoning." His first such work, *De natura rerum* ("On the Nature of Things") is an encyclopedic text, including theories about a wide range of subjects, including cosmology, time, and arithmetic (presumably *this* is the source of the "finger reckoning" which Eves mentions).

A later work, *De temporum ratione* ("On the Reckoning of Time") covers different calendars, the meaning of the Zodiac, and the calculation of Christian holy days such as Easter. As with some of Boethius' work, this was a part of the medieval European school curriculm of centuries after the death of Bede. Also like Boethius, Bede is more remembered for his non-mathematical work. His most famous work is *Historia ecclesiastica gentis Anglorum* ("Ecclesiastical History of the English People"), which was completed in 731. It tells the story of the establishment and spread of Christianity in England and the emergence of the Anglo-Saxon kingdoms. It survives in some 150 manuscripts (see The British Library website on *Historia ecclesiastica*; accessed 6/19/2023). Based on this work Bede is often called "The Father of English History." This historical information is based in-part on the Wikipedia webpage on Bede and on The British Library website on Bede (the second website is the source of the image below; both websites were accessed 6/19/2023).
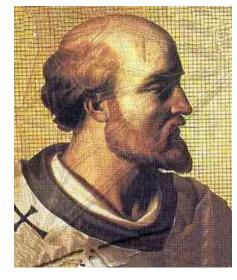

Note 8.1.D. Alcuin of York (735–May 19, 804) was an English mathematician and teacher. He attended school at Archbishop Ecgberht's School in York. After he completed his schooling there, he stayed as a teacher, and later became head-

master. From 781 to 796, Alcuin was headmaster of Charlemagne's Palace school in Aachen (in modern-day western Germany). While in this position, he developed the "Carolingian minuscule," a clear script which has become the basis of the way the letters of the present Roman alphabet are written. This impacted the history of math in that most of the mathematical works available (including some of those of the ancient Greeks) were copied into this new, more readable script during the 9th century. In 796, Alcuin became abbot of the Abbey of Saint Martin in Tours (France). Alcuin wrote elementary texts on arithmetic, geometry, and astronomy, written in a question-answer format. This work (though showing "little originality") influenced textbook writers for many centuries. Some of Alcuin's problems are given in Problem Study 8.1, parts (a) through (e). This history and the following image are from the MacTutor biography webpage of Alcuin (accessed 6/19/2023).

Note 8.1.E. We first encountered Gerbert of Aurillac (circa 946–May 12, 1003), also known as Pope Sylvester II, in Section 1.9. The Hindu-Arabic Numeral System. He was born in Auvergne (France) and educated in the Saint-Gerald monastery

there. After that, he spent three years at the cathedral school of Vich, near Barcelona (Spain). During this time, he learned the latest developments in math and astronomy of the Arabic world (likely by visiting Cordova and Seville at various times during these three years). After employment for two years as a tutor (as a former pope, his life is relatively well-documented), he settled in Rheims (France) and was put in charge of the cathedral school there. He made the school into a leading European center of learning. He wrote the texts which the students used. His math books covered geometry and computations with the Hindu-Arabic numerals (except zero), though he made no new mathematical discoveries. He taught the use of the abacus, terrestrial and celestial globes, and other astronomical instruments. Gerbert's mathematical impact is not in new contributions, but in the teaching of math and the popularization of the Hindu-Arabic numerals in Europe. Problem Study 8.1 parts (f) and (g) give a geometry problem from Gerbert's geometry book. In 991 he became Archbishop of Rheims, in 998 became the Archbishop of Ravenna (Italy), and on April 9, 999 he became pope. This history and the following image are from the MacTutor biography webpage of Gerbert (accessed 6/19/2023).