8.6. The Early Arithmetics

Note. This section considers the "arithmetic books" printed in Europe between 1578 and 1600. We mentioned such books in Supplement. Leonardo of Pisa (Fibonacci) and the *Liber abbaci*, where they were called "abbacus books." These books were needed (and written) because of the growth in an interest in education and the increase in commercial activity during the beginning of the Renaissance. Eves states (on page 267) that: "Three hundred such books were printed in Europe prior to the seventeenth century." His source on this seems to be David Eugene Smith's *Rara Arithmetica: A Catalogue of the Arithmetics Written Before the Year 1601 with a Description of Those in the Library of George Arthur Plimpton of New York* (Ginn and Company Publishers, 1908); this appears in Eves' bibliography for Chapter 8 (and Smith refers to the books as "arithmetics"). You can view it online (and download a copy in a variety of formats) at Archive.org (accessed 7/4/2023). *Rara Arithmetica* lists over 550 works, about 450 of which are classified as "arithmetics." It includes both "Printed Books" (in Part I on pages 3–429) and (handwritten) "Manuscripts" (in Part II on pages 433–494). These books fall into two categories: (1) those written in Latin by classical scholars, usually affiliated with a Church school, and (2) those written in the local language by teachers who were preparing boys for commercial careers (Eves, page 268).

Note 8.6.A. We now spend some time on another influential figure in the history of mathematics (as well as in mathematics education). David Eugene Smith (January 21, 1860–July 29, 1944) was an American mathematician born in Cortland, New York (near Syracuse in the Finger Lakes region); notice that he is one of the first Americans which has been mentioned in these notes (after Sequoyah in Supplement. Additional Numeral Systems, and Eric Temple Bell [who was actually born in Scotland, U.K.] and Talithia Williams in Section 3.2. Pythagoras and the Pythagoreans).

Smith got his primary education at Cortland Normal School and entered Syracuse University in 1877 (the university was only 6 years old at the time). He studied a variety of topics, including Latin, Greek, and Hebrew, and graduated in 1881 with a Bachelor of Philosophy degree. He traveled, studied law, and took graduate courses at Syracuse in history, modern languages, and mathematics, earning a Master of Philosophy in 1884. He accepted a teaching job at Cortland Normal School and continued graduate studies at Syracuse at the same time. In 1887 he earned a Ph.D. in fine arts. He continued to teach until 1891 adding history of math to his teaching of the usual high school math classes. He then took the position of chair of mathematics at Michigan State Normal School at Ypsilanti (recall that ETSU started as East Tennessee State Normal School in 1911). In this position, he started writing textbooks and developing the ideas of mathematical education that brought him acclaim. His work includes the books are *Plane and Solid Geometry* (1895) and *Higher Arithmetic* (1897), and the papers "Sources for Study of the Trisection Problem" (1895), "History of Modern Mathematics" (1896), and "Arithmetic in Rural and Village Schools (1897). Between 1898 and 1901 he served as principal of the Brockport (New York) Normal School. During this time, he visited New York City and met the publisher and philanthropist George Arthur Plimpton (July 13, 1855–July 1, 1936); this is the same Plimpton of Section 2.6. Babylonia: Plimpton 322. Smith was impressed with Plimpton's textbook collection. When offered the chair in mathematics at Teachers College of Columbia University (in NYC) in 1901, he accepted. Smith and Plimpton became friends and Smith was granted access to Plimpton's collection. This explains the subtitle of Smith's 1908 *Rara Arithmetica*: "A Catalogue of the Arithmetics ... with a Description of Those in the Library of George Arthur Plimpton of New York." Smith himself collected together a significant private library, which he allowed the students in his History of Math seminar at Columbia University to access. He donated his collection to the university in 1931 (Plimpton donated his library to Columbia University in 1936). He retired from Columbia University in 1926. He continued to travel and collect manuscripts and books of importance in the history of mathematics. Other books he wrote include:

1. *The Hindu-Arabic Numerals*, with Louis Charles Karpinski (Ginn and Company, 1911); this can be read on the Project Gutenberg website.

2. *A History of Japanese Mathematics*, with Yoshio Mikami (Open Court Publishing, 1914); this can be read on Archive.org.

3. *Elements of Projective Geometry* with G. H. Ling and George Wentworth (Ginn and Company, 1922); the Hathi Trust Digital Library has a link to several online versions of this.

4. *Mathematics*, in the series "Our Debt to Greece and Rome," Michigan Historical Math Collection (Marshall Jones Company, 1923; this can be read on the University of Michigan Historical Math Collection webpage.

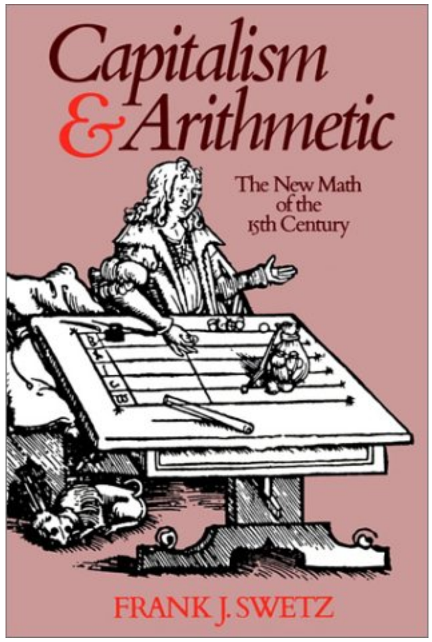
5. *History of Mathematics*, 2 Volumes (Ginn and Company, 1923/5); this was reprinted by Dover Publications in 1958 and Volume 1 can be viewed on Archive.org.

6. *A History of Mathematics in America before 1900*, with Jekuthiel Ginsburg; Carus Mathematical Monographs #5 (Mathematical Association of America, 1934); this can be viewed online on Archive.org.

This historical information in this note and the photo above are from the MacTutor biography of David Eugene Smith (each website of this note was accessed 7/4/2023).

Note 8.6.B. The earliest known printed abbacus book (or simply "arithmetic") is the *Treviso Arithmetic* (also known as *Arte del Abbaco* in certain library catalogues), printed on December 10, 1478 in Treviso, Italy (about 10 miles north of Venice). As such, it is the first printed book in the Western World devoted to mathematics (Eves, page 269). The author is unknown. *Treviso Arithmetic* is still in print as Frank J. Swetz, *Capitalism and Arithmetic: The New Math of the 15th Century—Including the Full Text of the Treviso Arithmetic of 1478*, translated by David

Eugene Smith, (Open Court, 1987). An extensive preview is available on Google Books (websites accessed 7/3/3023).

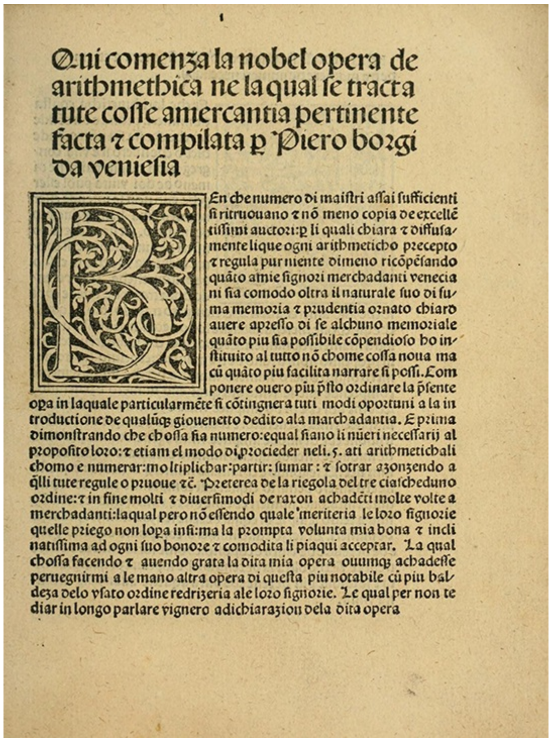


While browsing documents in the David Eugene Smith Collection at the library at Columbia University, Frank Swetz found 200 pages of typed and handwritten notes on "The Treviso Arithmetic 1478." Smith had completed his translation of the *Treviso Arithmetic* in 1907 and revised it in 1911. However, with the demands on his time, he never published the complete work (Swetz pages XIII and 37). Instead, he published some experts in "The First Printed Arithmetic (Treviso, 1478)," *Isis*, **6**, 311–331 (1924). This can be read online at the University of Chicago journals webpage (accessed 7/5/2023). George Plimpton had one of the few original copies of *Treviso Arithmetic* and this would be the copy on which Smith based his translation (Swetz also had access to this copy since is it part of the Plimpton Collection, Rare Books and Manuscript Library, of Columbia University). *Capitalism and Arithmetic* contains "David Eugene Smith: A Tribute" (pages 299–304); much of the MacTutor biographical webpage on Smith seems to be based on this source.

The seven chapters of Swetz's book covers general historical background (in his Chapter 1), gives Smith's translation of *Treviso Arithmetic* (in the 140 page Chapter 2), presents commentary on the contents of *Treviso* (Chapters 3 through 6), and considers the social, ecomonic, and commercial aspects of the contents of *Treviso* (Chapter 7). Swetz points out three reasons that *Treviso Arithmetic* is of historical importance. First, it is the earliest known dated, printed abbacus book (his pages XV and XVI). Second, it is one of the first books written for a popular audience (it is written in the Venetian vernacular), marking a turning point in the history of human knowledge. Third, it is an early example of the use of the Hindu-Arabic numerals in a time when the Roman numerals were still popular in much of Europe (though the Hindu-Arabic numerals had taken hold in the commercial activities of Italy at the time). *Treviso Arithmetic* covers the concepts and mechanics of early Renaissance arithmetic using the Hindu-Arabic numerals. It covers addition, subtraction, and division. For division, the quotient and remainder are computed. The computational algorithms are used in solving problems of commercial arithmetic. Though *Treviso* was first in print, it did not have a large impact.

Note 8.6.C. More influential than the *Treviso* was a 1484 abbacus book by Piero Borghi (circa 1424–circa 1494). Frank J. Swetz has two brief articles, "Mathematical Treasure: Borghi's Best Commercial Arithmetic," *Convergence* (May 2017), and "Mathematical Treasure: The Arithmetic of Piero Borghi," *Convergence* (August 2019)on this work; Swetz's articles are online on the *Convergence* website: Best Commercial (2017) and The Arithmetic (2019) (accessed 7/5/2023). Swetz described Borghi's book as "the first influential commercial arithmetic book of the

15th century. The book was untitled, nor did it have a title page. It is commonly known as *Qui comenza la nobel opera de arithmetica*, after the first words of the text: "Here begins a noble work on arithmetic…" (see below left).

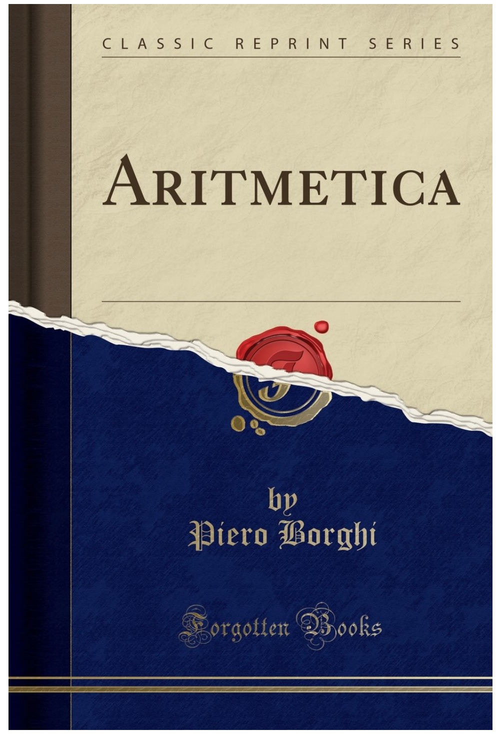


Qui comenza la nobel opera de arithmethica ne laqual se tracta tute cosse amercantia pertinente facta ⁊ compilata p Piero borgi da veniesia

BEn che numero di maistri assai sufficienti si ritruouano ⁊ nõ meno copia de excellẽtissimi auctori: p li quali chiara ⁊ diffusamente lique ogni arithmeticho precepto ⁊ regula pur niente dimeno ricõpẽsando quãto amie signori merchadanti venecia ni sia comodo oltra il naturale suo di suma memoria ⁊ prudentia ornato chiaro auere apresso di se alchuno memoriale quãto piu sia possibile cõpendioso ho instituito al tutto nõ chome cossa noua ma cũ quãto piu facilita narrare si possi. Componere ouero piu p̃sto ordinare la p̃sente opa in laquale particularmẽte si cõtingnera tuti modi oportuni a la introductione de qualũq; giouenetto dedito ala marchadantia. E prima dimonstrando che chossa sia numero: equal siano li nũeri necessarij al proposito loro: ⁊ etiam el modo di procieder neli .5. ati arithmetichali chomo e numerar: moltiplichar: partir: sumar: ⁊ sotrar azonzendo a q̃lli tute regule o pruoue ⁊c̃. Preterea de la riegola del tre ciascheduno ordine: ⁊ in fine molti ⁊ diuersimodi de raxon achadẽti molte volte a merchadanti: laqual pero nõ essendo quale meriteria le loro signorie quelle priego non lopa insi: ma la prompta volunta mia bona ⁊ inclinatissima ad ogni suo honore ⁊ comodita li piaqui acceptar. La qual chossa facendo ⁊ auendo grata la dita mia opera ouunq; achadesse peruegnirmi a le mano altra opera di questa piu notabile cũ piu baldeza delo vsato ordine redrizeria ale loro signorie. Le qual per non tediar in longo parlare vignero adichiarazion dela dita opera

Qui comenza la nobel…

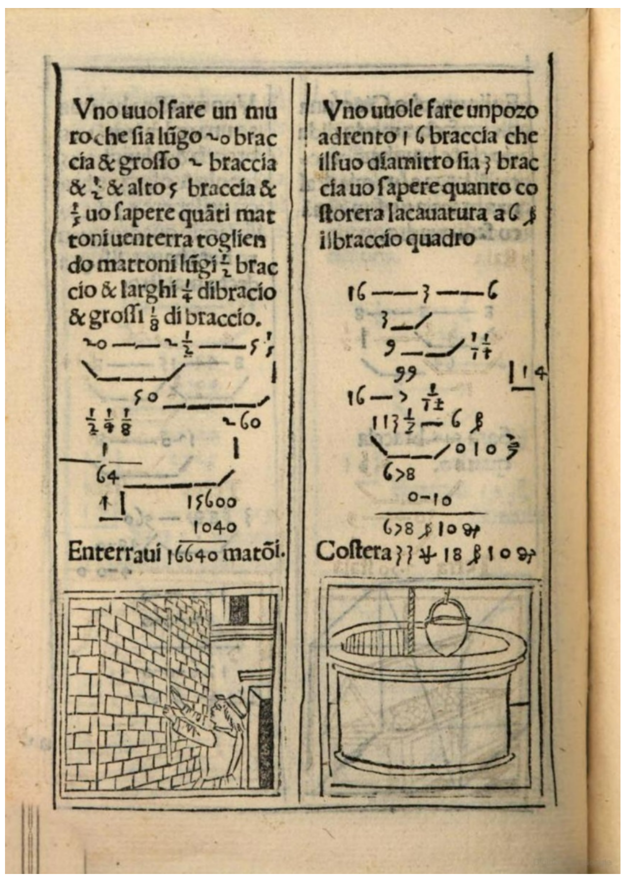
from Best Commercial…



From Amazon.com

An Italian version is still available as *Aritmetica by Peiro Borghi*, Classic Reprint Series (Forgotten Books, 2018). See above right. Nothing is know about Peiro Borghi, other than he is from Venice, so the MacTutor biography page on Borghi, on which the remiander of this note is based, describes the contents of his book. The MacTutor page is based on David Eugene Smith's "The First Great Commercial Arithmetic," *Isis*, **8**(1) 41–49 (1926) (available online on the University of Chicago journals webpage; accessed 7/5/2023). Borghi's book is also known as *Nobel opera de arithmetica*, *Libro dabacho*, and *Libro de Abacho*. Evidence of the influence of the work is the fact that it went through 16 editions between 1484 and 1577. Smith's paper mentions that George Plimpton's personal library contained several editions of the book and likely these are the sources for Smith's paper. Borghi did not aim his book at scholars, but instead at the merchant apprentice of time. The book begins by stating that it is not concerned with the special numbers of the Greeks (such as perfect numbers, abundant numbers, prime numbers, etc.), but instead only with numbers which are important to merchants. He first explains multiplication, then division, and later addition and subtraction (so some background in elementary computation is assumed). First he considers these operations for whole numbers and then for fractions (and explains reduction of fractions to lowest terms). He covers the Rule of Three (in which three numbers are related to an unknown through the four arithmetic "operations"), giving examples related to business. An easy example is: "If 4 1/2 yards of cloth cost 17 soldi, how much will 8 yards cost?" This is followed by a chapter on bartering, and then one on alloys (used to deal with conversions of different currencies based on the metals of which the currency is composed). The last chapter contains problems based on the techniques developed. An example from the last chapter is: "If 100 lire of Modon are equal to 1415 lire of Venice, and 180 lire of Venice are equal to 1850 lire of Corfu, and 240 lire of Corfu are equal to 360 lire of Negroponte how many lire of Modon are equal to 850 lire of Negroponte?" In modern terminology, you would describe this as a units conversion problem which you would address with conversion factors (see my online notes for Technical Physics 1 [PHYS 2110] on Chapter 1. Measurement; notice Note 1.1.A).

Note 8.6.D. Eves mentions a "less important" work by Filippo Calandri which was published in Florence (Italy) in 1491. The title is given as *Pitagora aritmetice introductor* (Italian;"Pythagorean Introduction to Arithmetic") in Keith Devlin's *The Man of Numbers: Fibonacci's Arithmetic Revolution* (Walker and Company, 2011), and Google Books gives it as *De Arithmetica opusculum* (Latin; "Essay on Arithmetic"). The book is famous for three things: (1) it is the first time the word "zero" appears in *print* (according to page 59 of Smith and Karpinski's *The Hindu-Arabic Numerals*, mentioned above), (2) it contains examples of our modern process of long division (Eves, page 269), and (3) it is the first illustrated math book published in Italy (Eves, page 269).

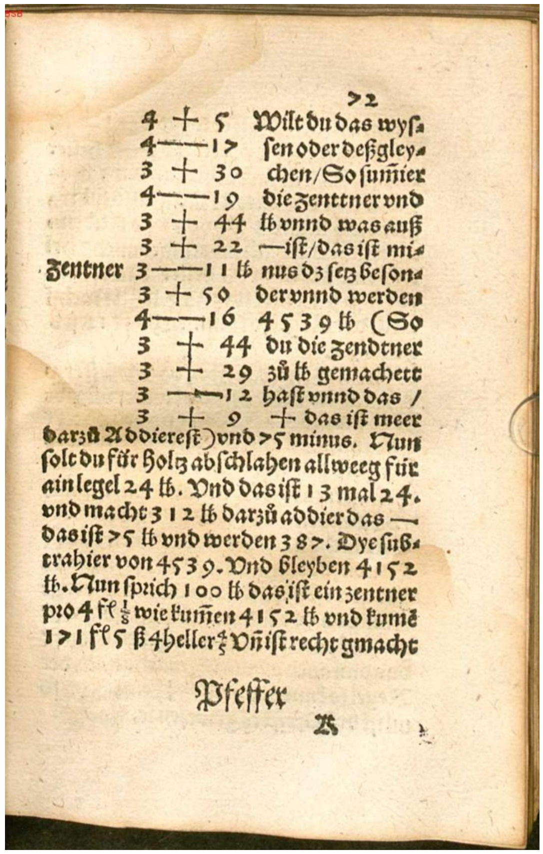


A copy of the book in Italian can be viewed on Google Books (accessed 7/6/2023). You can also download a PDF of the book from this sight (though it is not a clear copy). A page from this website showing an illustrated problem is given above (this corresponds to page 195 of the PDF file). Examples of our modern long division are said to be present, but I dare you to find them! (I suspect it is the modern *process* and not the modern *notation* that is given.)

Note. Chronologically, Luca Pacioli's (1445–1517), *Summa de arithmetica, geometria, proportioni et proportionalita* ("Summary of Arithmetic, Geometry, Proportion, and Proportionality") of 1494 should be listed here, but we considered it in detail in Section 8.5. The Fifteenth Century.

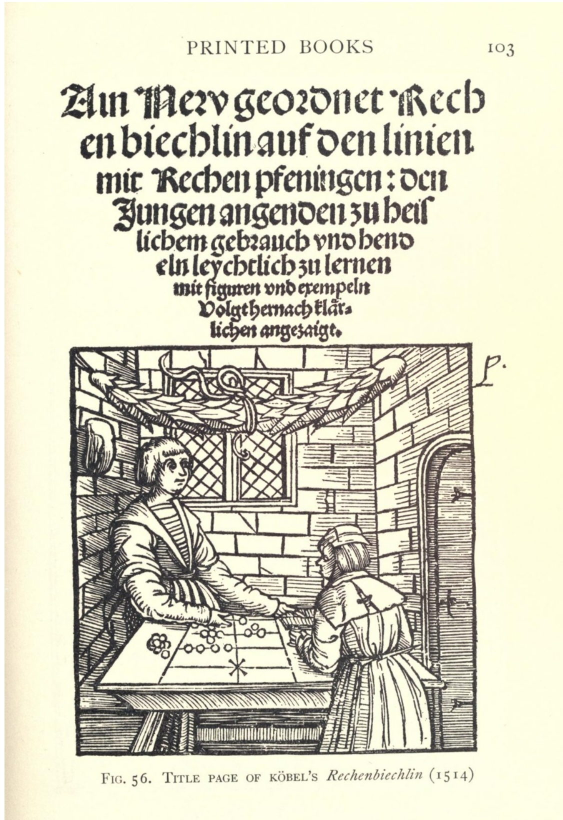
Note 8.6.E. We next consider abbacus books in Germany. We have already mentioned Johannes Widman (1462–1498) in Section 8.5. The Fifteenth Century; see Note 8.5.G. Widman attended the University of Leipzig (Germany), earning his first degree in 1482 and a master's degree in 1485. He then taught math at the University of Leipzig, covering arithmetic, computation on lines, and algebra. His 1486 algebra lecture was the first given in Germany and a notebook of a student who attended still survives! He covered fractions, proportions, computation with irrational numbers, and polynomials. He published *Behende und hupsche Rechnung auf allen kauffmanschafft* ("Numble and Smart Calculations for all Merchants") in 1489. It was written in German, and includes more examples with a wider range of applications than the previously available printed abbacus books. It consisted of three parts covering (1) counting with whole numbers, (2) proportions, and (3) geometry. Its main claim to fame lies with the fact that is introduces the symbols

"+" and "−" for positive and negative numbers. A 1526 version of the book can be viewed on Google Books (accessed 7/6/2023). The following image is from this website, and shows the + and − signs on page 72 of *Behende*:


72
4 + 5 Wiltu das wyſ-
4 — 17 ſen oder deßgley-
3 + 30 chen/So ſumier
4 — 19 die zentner vnd
3 + 44 lb vnnd was auß
3 + 22 — iſt/das iſt mi-
Zentner 3 — 11 lb nus dz ſetz beſon-
3 + 50 der vnnd werden
4 — 16 4539 lb (So
3 + 44 du die zendtner
3 + 29 zů lb gemachett
3 — 12 haſt vnnd das /
3 + 9 + das iſt meer
darzů Addiereſt) vnd 75 minus. Nun
ſolt du für holtz abſchlahen allweeg für
ain legel 24 lb. Vnd das iſt 13 mal 24.
vnd macht 312 lb darzů addier das —
das iſt 75 lb vnd werden 387. Dye ſub-
trahier von 4539. Vnd bleyben 4152
lb. Nun ſprich 100 lb das iſt ein zentner
pro 4 fl 1/5 wie kumen 4152 lb vnd kumē
171 fl 5 ß 4 heller 4/5 Vñ iſt recht gmacht
Pfeffer
B

The influence of Widman's work is reflected in the fact that is was reprinted in 1508, 1519, and 1526 in various parts of the German speaking world. This note is based on the MacTutor biography of Widman (accessed 7/6/2023).

Note 8.6.F. Jakob Köbel (1470–January 31, 1533) published an abbacus book in 1514 which ultimately went through at least 22 editions. Eves says that he was "a Rechenmeister of Heidelberg" (Eves, page 269); that is, he was a master calculator. He was also a printer, engraver, woodcarver, and public official. His abbacus book of 1514 was the *Rechenbüchlein* ("Arithmetic Booklet"), though it appeared under a number of other titles. The title page of the 1514 edition is given below.


PRINTED BOOKS 103

Ain New geordnet Rech
en biechlin auf den linien
mit Rechen pfeningen: den
Jungen angenden zu heil
lichem gebrauch vnd hend
eln leychtlich zu lernen
mit figuren vnd exempeln
Volgt hernach klär.
lichen angezaigt.

Fig. 56. Title page of Köbel's *Rechenbiechlin* (1514)

Köbel also wrote to other books, *Mit der Kryden* (on computational algorithms, 1515) and *Vysierbuch* (on measurements, 1520). His three books were sometimes published separately and sometimes together as a single book. The work is a commercial book in which all of the operations are performed by counters (as was still customary in most of Germany at the time). He covers the Rule of Three, inheritances, and exchange. He mostly uses Roman numerals, though the Hindu-Arabic numerals are introduced in a section on notation. A 1537 edition included all three of Köbel's books together. This edition formed one of the best books of the time and gave operations both with counters and with algorithms (in the "Mit der Kryeden" part). The usual business problems of the time are covered, and a chapter on the calendar is included (as required by the Church schools). Köbel also wrote a geometry book toward the end of his life. For more on this, see Frank Swetz and Victor Katz's "Mathematical Treasures—Jacob Kobel's Geometry," *Convergence* (January 2011) on the *Convergence* website (accessed 7/7/2023). This note and the image above are based on pages 100–114 of David Eugene Smith's *Rara Arithmetica* mentioned above and available online at Archive.org (accessed 7/4/2023).

Note 8.6.G. Adam Riese (December 23, 1492–March 30, 1559), or "Ries" or "Rieß," was born in Steffelstein (Germany, today called "Bad Staffelstein," near Nuremberg; sometimes he is called "Adam Rießvom Staffelstein"). Little is known of his early life, but in 1515 he moved to Annaberg (today, Annaberg-Buchholz in eastern Germany near the Czech boarder) where he studied algebra. In 1518 he moved to Erfurt (in the same region, near Dresden) where he ran a school as Rechenmeister ("master calculator"). He did not attend the University of Erfurt, but he did have contacts there and access to a collection of manuscripts on algebra and arithmetic which were donated to the university by Johannes Widman. There he wrote his first two books: *Rechnung auff der linihen* ("Reckoning with Lines," 1518) and *Rechnung auff der linihen und Federn* ("Reckoning with Lines and Springs," 1522). In 1523, he took an engineering job back in Annaberg and also taught math there. He completed a manuscript on algebra in Annaberg, *Coss*. He continued in various mining and engineering related positions between 1525 and 1539. As the reformation swept through the region, Riese (by this time a Lutheran) was well-positioned for advancement and he became a mathematician in the court of Annaberg. This position and the funds due to sales of his books allowed him to purchase the small Reisenburg Castle in Annaberg. The castle still stands and an image is given below right from the Google Maps website (accessed 7/7/2023).

Riese's first book, "Reckoning with Lines" (1518), gives instructions for using a lined calculating board (or abbacus). It is stated in the book that it is intended for children, so it was likely written to help in his teaching. His second book, "Reckoning with Lines and Springs" (1522), was the most influential of the German abbacus books. It went through over 100 editions. It was not meant for classroom use by children, but instead was intended for apprentice craftsmen. It covers the use of an abbacus and the use of the Hindu-Arabic numerals, so it covers both techniques which were is use in Europe at the time. Riese's third book was *Rechenung nach der lenge, auff den Linihen und Feder* ("Reckoning lengths with lines and springs," 1550); it is also commonly known as the *Practica* (the image of Riese given above appears in this book). Much of the writing of this work had been done in 1525, but printing costs delayed its publication. This was a textbook aimed at everyone, not just specialists and engineers. It describes addition, subtraction, multiplication, and, uncharacteristically for its time, division. Riese's algebra manuscript *Coss* (1525) was not put into print, other than in W. Kaunzner and H. Wussing (eds.), *Adam Ries, Coss* (B. G. Teubner Verlagsgesellschaft mbH, Stuttgart, 1992). This gives a facsimile of the manuscript published on the 500th anniversary of Riese's birth (in German). In the middle ages, the unknown was called "coss." (*Cosa* is Latin for 'thing.' In the 16th century, algebraists were called "cossists" and algebra was known as the "cossic art" for many years.) In *Coss*, Riese uses this term, along with radix, root, and thing for the unknown. It is a link between medieval descriptive algebra and the analytic algebra of today. Its status as an unprinted manuscript means that it had very little influence. Reise's fame, then, lies largely with his 1522 *Rechnung auff der linihen und Federn* ("Reckoning with Lines and Springs"). It does not appear that there is an English translation of it, but a German version can be found on the SLUB Dresden digital collection webpage (accessed 7/7/2023) The first page of the 1529 edition from this webpage is given below.


Rechnung auff
der Linihen vnd Federn/
Auff allerley handthirung gemacht/
durch Adam Rysen.

zum andern mal vber sehen/
vnd gemehrt.
Anno. M. D. xxix.

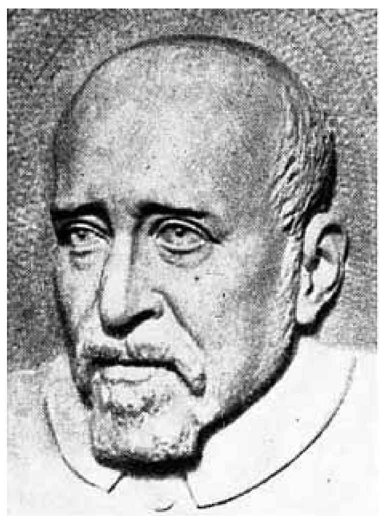
This note and the image of Riese above is from information on the MacTutor biography of Riese (accessed 7/7/2023).

Note 8.6.H. We conclude by considering two authors from the British Isle, Cuthbert Tunstall (1474–December 18, 1559)) and Robert Recorde (1510–1558). Tunstall (also spelled "Tonstall") authored the first printed book published in England that is devoted exclusively to mathematics (though it is written in Latin). Tunstall attended Balliol College, Oxford and King's Hall, Cambridge between 1491 and sometime after 1496. However, he did not finish a degree. Between 1499 and 1505 he studied law at Padua (Italy) and earned to degrees while becoming proficient in Greek, Latin, and mathematics. Back in England, he took a number of positions in the Church between 1505 and 1515. He then took on diplomatic roles (including for the court of Henry VIII) between 1515 and 1521. He became the Bishop of London in 1522. The is when he completed his abacus book, *De arte supputandi libri quattuor* ("On the art of arithmetic, book four," 1522). The book is heavily based on Luca Pacioli's (1445–1517) *Summa de arithmetica, geometria, proportioni et proportionalita* ("Summary of Arithmetic, Geometry, Proportion, and Proportionality") of 1494. We discussed Pacioli's *Summa* in some detail in Section 8.5. The Fifteenth Century (see Note 8.5.F). Tunstall made no claims of originality, so there are no actual concerns over plagiarism. A copy of the 1551 edition is available online on the Google Books webpage. A brief article on Tunstall's book is Frank Swetz's "Mathematical Treasure: Cuthbert Tunstall's De arte supputandi," *Convergence* (January 2013) available online on the *Convergence* webpage. It includes a page from the book that illustrates a lengthy long division computation. He performed diplomatic work between 1525 and 1527. He became Bishop of Durham in 1530. He had difficult relations with the government (due to his conservative leanings during the Reformation). He was arrested in 1550 and spent time in the Tower of London. He was removed from the position of bishop in 1552, reinstated in 1553, and removed again in 1559 (the year he died). His influence on English mathematics was recognized in 1533 when the first printed edition of Euclid's *Elements* in Greek (also the first edition to include diagrams) was dedicated to him "since he had explained the calculating of numbers in such an excellent manner." More details on this book are online at HistoryofInformation.com.

This note and the image of Tunstall above is from information on the MacTutor biography of Tunstall (the webpages of this note were accessed 7/8/2023).

Note 8.6.I. Robert Recorde (1510–1558) was a Welsh (i.e., from Wales in the U.K.) doctor and mathematician who wrote several books on mathematics and medicine. He is best known for his 1557 *The Whetstone of Witte* in which he introduced the equal sign "=." He graduated with a bachelor of art from the University of

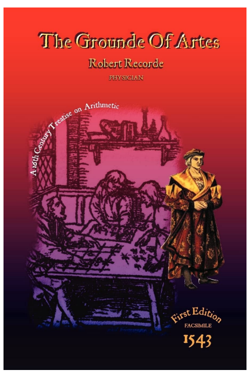
Oxford in 1531. In 1845 Recorde earned an M.D. degree from Cambridge, and he likely earned a master's of music from Oxford before this (but no record exists). He practiced medicine in London in the late 1540s, and became controller of the Bristol mint in 1549. In 1551 he was appointed by King Edward VI as the general surveyor of the mines and monies in Ireland, where he remained until 1553. Back in England, Recorde became involved in legal battles (especially with the Earl of Pembroke) related to conflicts between the Church of England (established by Henry VIII in 1534) and the Roman Catholic Church. In early 1557 Recorde was imprisoned, where he died in mid-1558.

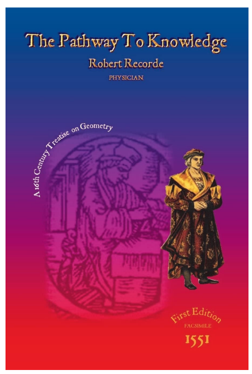


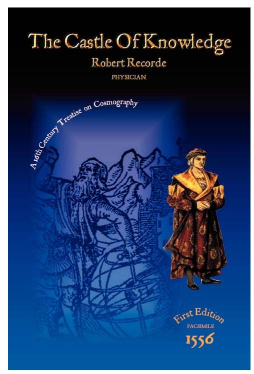
On the mathematical side, Recorde is the first to introduce algebra into England! He wrote many elementary textbooks, which he intended to be used both in the classroom and by anyone interested in the topic. All his books were written in English (as opposed to the more scholarly Latin), and most were in the from of a dialogue between a master and student. He published *The Grounde of Artes, teachings the Worke and Practise, of Arithmeticke, both in whole numbers and fractions* in 1543 (this is the only of his works mentioned by its title in this section of Eves). It was a commercial success, and covered Hindu-Arabic numerals, computation with counters, proportion, and the Rule of Three. It went through at least 29 printings (Eves, page 269). Recorde published an enlarged version which included rational numbers in general. In 1551 he published *Pathwaie to Knowledge, containing the First Principles of Geometry... bothe for the use of Instrumentes Geometricall and Astronomicall, and also for Projection of Plattes*, which covers the material of Euclid's *Elements*. No proofs are given, but instead explanations and examples are given to persuade students as to *why* the claims are true. In 1556, *The Castle of Knowledge, containing the Explication of the Sphere both Celestiall and Materiall, etc.* appeared giving an introduction to Ptolemy's version of astronomy, and hence covering the mathematics of the sphere. Copernicus' heliocentric theory (which Copernicus presented in his *De revolutionibus orbium coelestium* ["On the Revolutions of the Celestial Spheres"] in 1543, just before his death) is mentioned in *The Castle of Knowledge*, but Recorde says he does not believe the theory (historians are skeptical of this claim; at this point in history, advocacy of heliocentrism could lead to charges of heresy). *The Whetstone of Witte, whiche is the seconde parte of Arithmeteke: containing the extraction of rootes; the cossike practise, with the rule of equation; and the workes of Surde Nombers* was published in 1557, and it is this work in which Recorde introduces the equal sign, =. Whereas *The Grounde of Artes* gives an introduction to arithmetic, *The Whetstone of Witte* is a sequel considering the extraction of roots, arithmetic with irrational roots, and the theory of equations. In his consideration of quadratic equations, Recorde allows for negative coefficients, but does not allow for negative solutions. Of the four books mentioned in this note, each is still in print (in the form of facsimiles of the original editions) as follows:

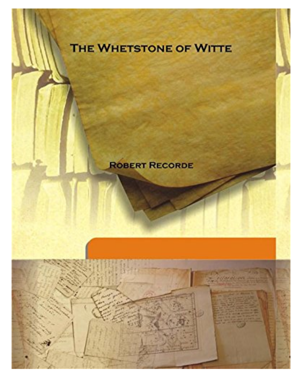
1. *The Grounde Artes* (CreateSpace Independent Publishing Platform, 2012),
2. *Pathway to Knowledge* (CreateSpace Independent Publishing Platform, 2013),
3. *Castle of Knowledge* (TGR Renascent Books, 2012), and
4. *The Whetstone of Witte* (Isha Books, 2013).

The following images of these four books are from the Amazon.com pages on Recorde's books.









Three of these (your humble instructor could not find the fourth) can be viewed online:

1. The 1618 edition of *The Grounde Artes* can be viewed on Google Books webpage (click on "Preview this book…"),
2. *Pathway to Knowledge* is on the Project Gutenberg webpage (and can be viewed online or downloaded), and
3. The 1557 edition of *The Whetstone of Witte* can be read online on the Archive.com webpage.

In the next section, we will explore Recorde's contribution to arithmetic notation. This note and the image of Recorde above is from information on the MacTutor biography of Recorde (the webpages of this note were accessed 7/8/2023).

Revised: 7/9/2023