Chapter 1. Numerical Systems
Study Guide

The following is a brief list of topics covered in Chapter 1 of Howard Eves' *Introduction to the History of Mathematics*, 6th Edition (Saunders College Publishing, 1990). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs, constructions, and examples given in class, and the homework problems.

Section 1.1. Primitive Counting.

Section 1.2. Number Bases.
Base (radix or scale) of a number system, Aboriginal Australian base 2 number system (or two-word additive system), quinary scale and the use of two hands to count base 5, duodecimal scale and its advantages and its remnants today, vigesimal scale and body counting and the Maya, the Celts and their base 20 influence on modern terms and language, sexagesimal scale and the Babylonians and its remnants today.

Section 1.3. Finger Numbers and Written Numbers. Numeral, the objects of mathematics are IDEAS, digits, The Venerable Bede and his *De ratione temporum*, the 16th century Persian dictionar *Farhangi Djihangiri*, the trivium (grammar, rhetoric, logic), the quadrivium (arithmetic, geometry, astronomy, music), Luca Pacioli and his "Summary of arithmetic, geometry, proportion, and proportionality" (1494), finger numbers, ASL numerals.

Section 1.4. Simple Grouping Systems.
Simple grouping system, Egyptian hieroglyphic numeral system, Egyptian hieratic script, cuneiform and how numerals are made, Babylonian numerals, the Babylonian subtraction symbol, examples of the use of Babylonian numerals, the Attic (or Herodianic) Greek numerals and examples, Roman numerals and examples.

Section 1.5. Multiplicative Grouping Systems.
Multiplicative grouping system and two sets of symbols, our numeral system as a multiplicative grouping system, the four main types of Chinese numerals (standard, official, commercial, and stick) and their dates, the oracle bones and the Shang system of numerals, the standard Chinese

numerals and examples, Japanese and Sino-Vietnamese numerals.

Section 1.6. Ciphered Numeral Systems.

Ciphered numeral system, the alphabetic Greek numeral system, digamma, koppa, sampi, examples of the alphabetic Greek representation, using primes M for larger numbers.

Section 1.7. Positional Numeral Systems.

Positional numeral system, digits, unique representation in a positional numeral system, our numeral system as an example of a positional numeral system, Babylonian representation of numbers greater than 60 and the alternating sequence of 1-for-10 and 1-for-6 replacements, the Babylonian zero (and its limitations), examples of Babylonian representations, the location and era of Maya civilization, history of Maya civilization, the Dresden Codex, the Maya numerals, replacement of a factor of 20 for a factor of 18 in the Maya positional numeral system and reasons for it, examples of the Maya representation, synodic period of a planet, the list of observations of Venus in the Dresden Codex.

Supplement. Additional Numeral Systems.

Aboriginal Australians' body counting and the "one, two, three, many" myth (and Alf Sommerfelt), Australian antrhopologist Alfred Howitt, visual counting, Mesopotamia, Fertile Crescent, Tigris and Euphrates Rivers, earliest writing (3500 BCE to 3000 BCE), Old Babylonian Empire, Roman use of *calculi*, Sumerians and clay counters (and dates), clay envelopes, Sumerian written numerals, Sumerian cuneiform, writing and the recording of thoughts, pictographs and symbols with phonetic meanings, Egyptian hieratic numerals and their use in adding, alphabetic Hebrew numerals, Sequoyah, Cherokee Nation, syllabary, Sequoyah's numerals and their use in adding, the Inuit Kaktovik numerals and their use in addition, subtraction, and division.

Section 1.8. Early Computing.

"Mental difficulties" and physical difficulties causing the delay of the spread of modern approaches to multiplication and division, papyrus and its construction and its dates, parchment and vellum, palimpsest, wax writing surface of the Romans, abax (sand tray), use of the abacus for adding Roman numbers.

Section 1.9. The Hindu-Arabic Numeral System. Johannes Gutenberg and the standardization of the Hindu-Arabic numerals, Indian numerals, the Brâhmî numerals, Sanskrit, Gupta numerals, Nagari numerals, zero and the *Lokavibhâga* (458 CE), the Bakshsali manuscript and carbon dat-

ing, Bhâskhara and his commentary on *Âryabhatîya* (629), Brahmagupta and his *Brâmasphutasidd-hânta* and his rule of signs, Arab advances in the 8th to the 13th century, Arabic translations of ancient Greek works, Abu al-Biruni, Ibn Khaldun and his *Muqakkimah*, Al-Khwarizmi (*Al jabr wa'l muqabala*/Transposition and Reduction, and *Kitab al hami' wa'l tafriq be hisab al hing*/Indain Technique of Addition and Subtraction), development of Indian numerals in the western and eastern parts of the Arabic world, ghubar numerals, the SLOW spread of Indian numerals through Europe, abacists and algorists, abacus (or "European abacus"), the Inquisition and knowledge suppression, Gerbert of Aurillac (Pope Sulvester II), the *Codex Vigilanus* as the first appearance of Hindu-Arabic numerals in "the west," Gerbert's abacus and its role in the spread of Hindu-Arabic numerals, the Crusades and their positive effect on spreading the Hindu-Arabic numerals, Leonardo of Pisa (Fibonacci) and *Liber abaci.*

Section 1.10. Arbitrary Bases. Representing a nonnegative integer in terms of a given base $b \geq 2$, duodecimal system and examples, use of the Division Algorithm in finding representations of numbers base b and examples, addition and multiplication tables and their use is more complicated additions and multiplications and division.

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