

Chapter 3. Pythagorean Mathematics

Study Guide

The following is a brief list of topics covered in Chapter 3 of Howard Eves' *Introduction to the History of Mathematics*, 6th Edition (Saunders College Publishing, 1990). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs, constructions, and examples given in class, and the homework problems.

Section 3.1. Birth of Demonstrative Mathematics.

Asia Minor, Miletus, Thales, writers who reference Thales (Pliny, Plutarch, Diogenes Laertius, Proclus), Thales' measurement of the height of the Egyptian pyramids, Plato's seven sages includes Thales, results credited to Thales (a circle is bisected by any diameter, the base angles of an isosceles triangle are equal, vertical angles formed by two intersecting lines are equal, and two triangles are congruent if they have two angles and one side equal, an angle inscribed in a semicircle is a right angle), Eudemus' *History of Geometry*, the "Eclipse of Thales," Aristotle's *Metaphysics* includes some of Thales' philosophy.

Supplement. Proclus's Commentary on Eudemus' *History of Geometry*

Eudemus of Rhodes and his lost *History of Geometry*, Proclus of Diadochus and his commentary on Euclid's *Elements*, Proclus' version of Eudemus' *History of Geometry*, Thales, Hippias of Elis, quadratrix, Pythagoras of Samos, Oenopides of Chios and his compass and straight edge ideas, Hippocrates of Chios, Theaetetus of Athens and his theory of irrationals, Eudoxus of Cnidus and his theory of proportions and his method of exhaustion, Eudoxus likely contribution to Euclid's Book XII, Menaechmus and the introduction of conic sections and the doubling of the cube, Dinostratus and squaring the circle, Theudius of Magnesia.

Section 3.2. Pythagoras and the Pythagoreans.

Euclid's *Elements* makes previous work on geometry obsolete, translators Johan Ludwig Heiberg and Sir Thomas Little Heath, Proclus' commentary on Euclid and the *Eudemian Summary*, Pythagoras of Samos, E. T. Bell's *Men of Mathematics*, Lynn Osen's *Women in Mathematics*, Claudia Henrion's *Women in Mathematics: The Addition of Difference*, Women and Mathematics Program (WAM), Talithia Williams' *Power in Numbers: The Rebel Women of Mathematics* (Grade Hopper, Katherine Johnson, and Ingrid Daubechies), Proclus' comments about Pythagoras, biographies of Pythagoras (by Diogenes Laertius, Porphyry of Tyre, Iamblichus of Chalcis, and Photius), Pythagorean mysti-

cism as outlined in Mario Livio's *Is God a Mathematician?*, the biography of Pythagoras as given in Jason Socrates Bardi's *The Fifth Postulate: How Unraveling a Two-Thousand-Year-Old Mystery Unraveled the Universe*.

Section 3.3. Pythagorean Arithmetic.

The natural numbers \mathbb{N} , divisors and proper divisors of natural numbers, number theory, Iamblichus credits the Pythagoreans with the discovery of amicable numbers, the history of amicable numbers and Arab mathematicians, Walter Rouse Ball's *A Short Account of the History of Mathematics*, perfect numbers, deficient numbers, abundant numbers, Mersenne primes, Nichomachus and his *Introductio Arithmetica*, perfect numbers and Mersenne primes, unsolved problems concerning perfect numbers and Mersenne primes, k -tuply perfect numbers, superabundant numbers as defined by Alaoglu and Erdős, figurate numbers (triangular numbers, square numbers, pentagonal numbers), Aristotle's comments in *Metaphysics* on the Pythagoreans and numbers and music and figurate numbers, any square number is the sum of two successive triangular numbers (Theorem I), the n th pentagonal number is equal to n plus three times the $(n - 1)$ th triangular number (Theorem II), the sum of any number of consecutive odd integers is a perfect square (Theorem III).

Section 3.4. Pythagorean Theorem and Pythagorean Triples.

The Pythagorean Theorem was known before Pythagoras, Pythagoras likely used a dissection proof such as those given in Figure 13, Thomas Heath's comments on the Pythagorean Theorem's origin in *A History of Greek Mathematics*, Pythagorean triple.

Section 3.5. Discovery of Irrational Magnitudes.

Commensurable and incommensurable lengths, rational numbers are ratios, construction of rational numbers given a segment of unit length, decimal expansions of rational and irrational numbers, the argument that $\sqrt{2}$ is irrational, Thomas Heath's quote of Aristotle concerning the Pythagoreans and the irrationality of $\sqrt{2}$, Theodorus of Cyrene and the construction of other irrationals, Julian Havil's book *The Irrationals* and his comments on Hippasus, Eudoxus' new theory of proportions as mentioned in Heath's Introductory Note to Book V of Euclid.

Section 3.6. Algebraic Identities.

Geometrical algebra, Pythagoras and the origin of geometrical algebra, the wording of claims in Euclid's *Elements*, $(a + b)^2 = a^2 + 2ab + b^2$, Euclid's Proposition II.5 and its proof, $4ab + (a - b)^2 = (a + b)^2$, $(a + b)(a - b) = a^2 - b^2$, Euclid's Proposition II.6.

Section 3.7. Geometric Solution of Quadratic Equations.

Method of proportions, $x^2 = ab$ and the associated construction, applying a parallelogram to a line segment, an applied parallelogram falling short, an applied parallelogram exceeding, Euclid's Proposition I.44 (Note 3.7.A), Euclid's Proposition VI.28 and the construction associated with $x^2 - ax + b^2 = 0$ (Note 3.7.B), Euclid's Proposition VI.29 and the construction associated with $x^2 - ax - b^2 = 0$ (Note 3.7.C), the construction associated with $x^2 - ax + b^2 = 0$ and $x^2 + ax + b^2 = 0$ when there are two positive or two negative solutions (Proposition VI.28 again; Note 3.7.D), the construction associated with $x^2 + ax - b^2 = 0$ and $x^2 - ax - b^2 = 0$ when there is one positive solution and one negative solution (Proposition VI.28 again; Note 3.7.E), compass and straight edge constructions as addressed in modern algebra, constructible numbers.

Section 3.8. Transformation of Areas.

Rectilinear angle and figure, Euclid's constructions of objects of a given area (Propositions I.42, I.44, I.45, II.14), stories related to the Pythagoreans and the sacrifice of an ox, construction of a triangle with an area equal to that of a given rectilinear figure.

Section 3.9. The Regular Solids.

Regular polyhedron, the five regular polyhedra (tetrahedron, hexahedron [i.e., cube], octahedron, dodecahedron, and icosahedron), Euclid's constructions of the five regular solids in the *Elements* (Propositions XIII.13, XIII.14, XIII.15, XIII.16, XIII.17), the concluding result of Euclid's *Elements* (that there are only five regular solids), the Pythagoreans and the association of the five regular solids with the Greek elements (earth, fire, air, water, and aether), the spread of the idea of the association of the five regular solids with elements (by Empedocles, Plato, and Aristotle), Plato's constructions of the five regular solids given in *Timaeus*, Iamblicus's biography of Pythagoras in which he references Hippasus to the dodecahedron, Kepler's failed attempt to use the regular solids in a model of the movement of the planets.

Section 3.10. Postulational Thinking.

Postulational method, "axiom" and "postulate" mean the same thing in modern terminology, Aristotle's *Posterior Analytics* and his comments on first principles and the difference between a hypothesis and a postulate, David Hilbert and a modern axiomatic approach to geometry.

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