Chapter 4. Duplication, Trisection, and Quadrature

Study Guide

The following is a brief list of topics covered in Chapter 4 of Howard Eves' *Introduction to the History of Mathematics*, 6th Edition (Saunders College Publishing, 1990). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs, constructions, and examples given in class, and the homework problems.

Section 4.1. The Period from Thales to Euclid.
Bronze Age Mycennaean Greece, Greek Dark Ages/Homeric Ages, Ancient Greece, the periods of Greek history, city states, Magna Graecia, Anotolia, Ionia, Aegean Sea, Ionian Sea, Ionian school of Pre-Socratic philosophy, Anaximander, Classical Period (Note 4.1.B), Socrates/Plato/Aristotle, Parthenon, Persian War, first Persian invasion of Greece, Battle of Marthon, Battle of Thermopylae ("300 Spartans"), Battle of Mycale, Delian League, Peloponnesian War, Pericles, siges of Athens, Athens plague of 430 BCE, Peace of Nicias, second stage of the Peloponnesian War, Athens surrender, Archytas, Plato and his Academy (Notes 4.1.E and 4.1.F), Platonism, formalism, David Hilbert, Note 4.1.G (Eudoxus of Cnidus, Menaechmus, Dinostratus, Theaetetus, Aristotle and Aristotelean logic).

Section 4.2. Lines of Mathematical Development.
The 300 years preceding Euclid's *Elements*, the line that leads to Euclid's *Elements* (Pythagoras, Hippocrates, Theodorus, Theaetatus, Eudoxus), the line to calculus concepts (Elea, Antiphon the Sophist, Eudoxus, Democritus of Abdera), the line that leads to higher geometry.

Section 4.3. The Three Famous Problems.
Duplication ("doubling") of the cube, trisection of an angle, quadrature of the circle, "The energetic search for solutions to these three problems profoundly influenced Greek geometry and led to many fruitful discoveries, such as that of the conic sections, many cubic and quartic curves, and several transcendental curves" (Eves, page 110).

Section 4.4. The Euclidean Tools.
Compass and straight edge, Euclid's first two postulates and the straight edge, Euclid's third postulate and the compass, Euclidean tools, ruler, modern compass, Oenopides makes compass and straight-edge constructions central to geometry (maybe), the proof of Euclid's Proposition I.1 (Note 4.4.A), the constructible numbers and quadratic space/plane (continuity is not needed to do the compass and straight edge geometry of Euclid; Note 4.4.B).

Section 4.5. Duplication of the Cube.
"Doubling" the cube, $\sqrt[3]{2}$ is not constructible, history of the duplication of the cube (King Minos and the tomb for his son Glaucus), Hippocrates and computations of mean proportionals (Note 4.5.A), Menaechmus and his use of conic sections to find $\sqrt[3]{2}$ (Note 4.5.B), Apollonius, Descartes, conic sections in terms of focus/directrix/eccentricity, Plato's mechanical method (Note 4.5.C).

Section 4.6. Trisection of an Angle.
Trisection of *any* angle, impossibility of the trisection of a $60°$ angle and $\cos(20°)$ is not a constructible number, *A Budget of Trisections* by Underwood Dudley, trisection of an angle as a verging problem (Figure 31).

Section 4.7. Quadrature of the Circle.
Doubling the cube (or "quadrature of the cube") is equivalent to constructing π which was shown to be impossible by Pierre Wantzel and Ferdinand von Lindemann (Note 4.7.A), constructible numbers, algebraic numbers, transcendental numbers, Anaxagoras and the squaring of the circle, Hippocrates of Chios and his lunes, the quadratix of Hippias and the squaring fo the circle (Note 4.7.B), Archimedes and the use of the Archimedean spiral to trisect an angle (Note 4.7.B), the Archimedean spiral and squaring the circle.

Section 4.8. A Chronology of π.
References on π (Herman Schepler's "The Chronology of Pi," David Blatner's *The Joy of* π, and *Pi: A Source Book* by Breggren, Borwein, and Borwein), weird history of π ($\pi = 3$ based on 1 Kings 7:23 of the Bible and the Indiana General Assembly contemplation of setting π to 3.2), the area of a circle is proportional to the square of its radius (Euclid's Proposition XII.2), definition of π, the area of a circle is one-half of the circumference times the radius (Archimedes' Proposition 1 in *Measurement of a Circle* and its proof by the method of exhaustion), Archimedes' proof that $3\frac{10}{71} \leq 3\frac{1}{7}$ using a 96-gon (in Proposition 3 of *Measurement of a Circle*; Note 4.8.B), Claudius

Prolemy and the *Almagest*, Zu Chongzhi's approximation of π as $355/113 \approx 3.1415929$, Aryabhata the Elder and his approximation of π as $3927/1250 = 3.1416$

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