

Chapter 5. Euclid and His Elements

Study Guide

The following is a brief list of topics covered in Chapter 5 of Howard Eves' *Introduction to the History of Mathematics*, 6th Edition (Saunders College Publishing, 1990). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs, constructions, and examples given in class, and the homework problems.

Supplement. Euclid's Elements-A 2,500 Year History.

Sir Thomas Heath's translations of the *Elements*, Babylonian cuneiform tablets, Plimpton 322, Egyptian geometry driven by the flooding of the Nile River, the Rhind Mathematical Papyrus, Thales of Miletus, measurement for the heights of the pyramids of Giza, Pythagoras and the Pythagorean school, Hippocrates of Chios, lunes of Hippocrates, *reductio ad absurdum*, the University of Alexandria and the Alexandrian Library, Proclus' biography of Euclid in his commentary on the *Elements*, topics of the 13 books of the *Elements*, other works of Euclid, Heron of Alexandria, Eudemus of Rhodes' history of geometry, Proclus' commentary on Book I, Archimedes and his *Method*, Apollonius and his *Conics*, Hypatia, preservation of the classical works in the Arabic world, an-Nairizi, Abu al-Bujani, al-Kindi, Thabit ibn Qurra, al-Hasan al-Haitham, Nasiraddin al-Tusi, Bodleian manuscript of the *Elements* (888 CE), Vatican Manuscript Number 190 of the *Elements*, the Oxyrhynchus papyrus, the first printed version of the *Elements* (1482, Venice), Henry Billingsley and the first English translation of the *Elements* (1570), Robert Simson's English version of the *Elements*, John Playfair's edition of the *Elements* and Playfair's Theorem, Johan Ludvig Heiberg and his translations (of Archimedes, Apollonius, and Euclid), Thomas Heath and his translations into English (of Diophantus, Apollonius, Archimedes, Aristarchus, and Euclid), copies of the *Elements* in local libraries, the Dover editions, the Encyclopedia Britannica's *Great Books of the Western World*, the *Elements* online and Davis Joyces's online version of the *Elements*.

Section 5.1. Alexandria.

Corinthian War, Treaty of Antalcidas, Battle of Leuctra, Thebans, Third Sacred War, Phillip II of Macedon, League of Corinth, Alexander the Great (Alexander III of Macedonia), Aristotle tutored Alexander, Alexander's invasion of the Middle East and northeast Africa, the founding of Alexandria in Egypt, Alexander's campaign in Asia/India, the death of Alexander, Alexander's role in the founding of Alexandria, Dinocrates of Rhodes, Cleomenes, Seleucid Empire, Ptolemaic Kingdom, Ptolemy I Soter, Alexander's tomb, "Royal Road to geometry," Mouseion of Alexandria,

Library of Alexandria, Lighthouse of Alexandria, the business of the Mouseion.

Section 5.2. Euclid.

The biographical information on Euclid as given by Proclus Diadochus in his *A Commentary on the First Book of Euclid's Elements* (Note 5.2.A), Jean Itard's possible scenarios of Euclid (he was an historical character, he was the leader of a mathematical team, and he was not an historical character and the *Elements* were written by a team and credited to a nonexistent author; Note 5.2.B), other books by Euclid (*The Data*, *On Divisions*, *The Porisms*, *The Conics*, *The Phaenomena*, *The Optics*, *Elements of Music*).

Section 5.3. Euclid's "Elements".

The most influential book on mathematics, the *Elements* were so complete that traces of earlier geometric works faded away, Heron of Alexandria's commentary on the *Elements*, Theon of Alexandria's commentary (in Greek), Vatican Manuscript #190 of the *Elements*, "Books XIV and XV" of the *Elements* by Hypsicles of Alexandria and Isidore of Miletus, Proclus *Commentary* on Book I, Proclus' biography (Note 5.3.B), Pappus of Alexandria and his *Mathematical Collection* (Note 5.3.C), Papyrus Oxyrhynchus 29 (Note 5.3.D), Bodelian manuscript of the *Elements* (Note 5.3.E), other versions of the *Elements* (the Laurentian Library copy, Manuscripts 18 and 19 of the Communal Library in Bologna, the Viennese Manuscript, Paris Manuscripts 2466 and 2344), the first printed version of the *Elements* in 1482, Arabic translators and commentators on the *Elements* (Abu'l Abbas A;-Fadl, Mohammas Abu'l Wafa, Abu Al-Kindi, Thabit Qurra, al-Hasan al-Haytham, Nasir al-Din Al-Tusi), Billingsley's 1570 first English translation and "Book XVI" (Note 5.3.G), other English translations (Issac Barrow, Robert Simson), John Playfair 1795 edition and Playfair's Axiom (Note 5.3.I), negation of Playfair's Axiom and non-Euclidean geometry (elliptical and hyperbolic geometry), Johnna Heiberg and his work and biography (Note 5.3.J), Thomas Heath and his work and biography (Note 5.3.K), Dover Publications editions of Heath's version of *Elements*, the *Elements* online (Note 5.3.L).

Section 5.4. Content of the "Elements".

Areas covered in the *Elements*, Book I definitions (Note 5.4.A), Book I postulates (Note 5.4.B), Parallel Postulate (Note 5.4.C), interior angles, Book I Common Notions, Book I coverage (triangles, and congruence, theory of parallels, parallelograms triangles, squares), the Pythagorean Theorem and its converse, neutral geometry, transformational geometry and Proposition I.4, *pons asinorum* (Proposition I.5, Note 5.4.E), proof by contradiction (Note 5.4.F), Book II (Note 5.4.G), Book III

(circles, angles, chords; Note 5.4.H), Book IV (regular polygons; Note 5.4.I), Book V and Eudoxus' theory of proportion (Note 5.4.J), Dedekind cut, Book VI (theory of proportion applied to geometric figures; Note 5.4.K), similar triangles, mean proportional/geometric mean, Proposition VI.31 and its relation to the Pythagorean Theorem, lunes of Hippocrates, elementary number theory (Books VII, VIII, and IX; Note 5.4.L), The Euclidean Algorithm, Book VIII (Note 5.4.M), Book IX (Note 5.4.N), The Fundamental Theorem of Arithmetic, there is an infinite number of primes and its proof (Proposition IX.20), sum of a geometric progression (Proposition IX.35), Book X (Note 5.4.O), commensurable, incommensurable, commensurable in square, irrational, magnitude, method of exhaustion, Pythagorean triple, Books XI, XII, and XIII (solid geometry, definitions, propositions; Note 5.4.P), Book XIII and regular polyhedra, pyramid (tetrahedron; Proposition XIII.13), octahedron (Proposition XIII.14), cube (Proposition XIII.15), icosahedron (Proposition XIII.16), dodecahedron (Proposition XIII.17), Euclid's "Concluding Remark," the myth that the *Elements* were written to prove there are exactly 5 Platonic solids (Note 5.4.R).

Section 5.5. The Theory of Proportion.

Proposition VI.1, Pythagorean "proof" of Proposition VI.1 (Note 5.5.A), Euclid's proof of Proposition VI.1 (Note 5.5.B), modern analytic proof of Proposition VI.1 (Note 5.5.C).

Section 5.6. Regular Polygons.

Construction of regular n -gons in the *Elements* ($n \in \{3, 4, 5, 6, 15\}$), the use of bisection of angle to construct regular n -gons, Gauss and the construction of a regular 17-gon, Pierre Wantzel, Fermat prime, classification of constructible regular n -gons (the Gauss and Wantzel Theorem), construction of a regular 257-gon, construction of a regular 65,537-gon.

Section 5.7. Formal Aspect of the "Elements".

Definition/postulate/theorem/proof approach, undefined terms, axiomatic systems, postulates versus axioms, Playfair's Axiom, non-Euclidean geometry, consistent/independent/complete axiomatic systems, flaws in the *Elements*, synthetic and analytic approaches to geometry,

Section 5.8. Euclid's Other Works.

Data and surviving copies (Note 5.8.A), *On Divisions of Figures*, Proclus commentary, Heath's history of copies of *On Divisions of Figures* (Note 5.8.B), the *Phaenomena* and spherical geometry (Note 5.8.C), *On the Moving Sphere*, *Optics* (Note 5.8.D), the *Porisms* and Pappus (Note 5.8.E), the *Pseudaria* and *Surface-loci* (Note 5.8.F).