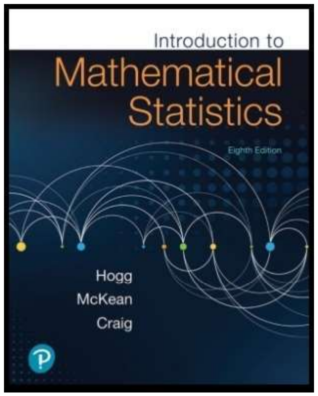
Mathematical Statistics 1

Chapter 3. Some Special Distributions

3.1. The Binomial and Related Distributions—Proofs of Theorems



Theorem 3.1.1

Theorem 3.1.1. Let $X_1, X_2, \ldots, X_m$ be independent random variables such that X_i has the binomial $b(n_i, p)$ distribution for $i \in \{1, 2, \ldots, m\}$. Let $Y = \sum_{i=1}^m X_i$. Then Y has a binomial $b\left(\sum_{i=1}^m n_i, p\right)$ distribution.

Proof. As shown in Note 3.1.A, the moment generating function of X_i is $M_{X_i}(t) = (1 - p + pe^t)^{n_i}$. Since the X_i are independent, the hypotheses of Theorem 2.6.1 are satisfied and it implies

$$M_Y(t) = \prod_{i=1}^m (1 - p + pe^t)^{n_i} = (1 - p + pe^t)^{\sum_{i=1}^m n_i}.$$

This is the moment generating function of the binomial $b\left(\sum_{i=1}^m n_i, p\right)$ distribution, as claimed. □