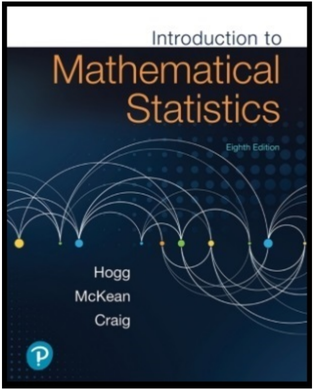
Mathematical Statistics 1

Chapter 3. Some Special Distributions

3.1. The Binomial and Related Distributions—Proofs of Theorems



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Proof. As shown in Note 3.1.A, the moment generating function of X_i is $M_{X_i}(t) = (1 - p + pe^t)^{n_i}$. Since the X_i are independent, the hypotheses of Theorem 2.6.1 are satisfied and it implies

$$M_Y(t) = \prod_{i=1}^m (1 - p + pe^t)^{n_i} = (1 - p + pe^t)^{\sum_{i=1}^m n_i}.$$

This is the moment generating function of the binomial $b\left(\sum_{i=1}^m n_i, p\right)$ distribution, as claimed. □

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