Chapter 1. Probability and Distributions

Section 1.1. Introduction

Note. This section intuitively introduces probability without any concern about the philosophical meaning. This requires the performance (in practice or in theory) of an experiment with a set of possible outcomes. We do not know the outcome of the experiment with certainty prior to the performance of the experiment.

Definition. If an experiment can be repeated under the same conditions it is a *random experiment.* The set of every possible outcome is the *sample space*, denoted $\mathcal{C}$.

Example 1.1.2. Two dice, one red and one white, are rolled. If we assume that the dice may be repeatedly rolled under the same conditions then this is a random experiment. The sample space consists of the 36 ordered pairs:

$$\mathcal{C} = \{(x, y) \mid x, y \in \{1, 2, 3, 4, 5, 6\}\}.$$

Note/Definition/Example. Notationally, we denote the elements of the sample space with lower case letters such as a, b, c. Subsets of the sample space are *events* and we denote them with upper case letters such as A, B, C. In the previous example, the event that the sum of the dice is 7 is

$$B = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}.$$

Definition. If an experiment is performed N times and a specific event occurs f times, then f is the *frequency* of the event and f/N is the *relative frequency* of the event.

Note. If for N large, the relative frequency of an event approaches a value p, then p is the "probability" of the event. This is the "frequency interpretation" of probability or the "relative frequency approach."

Example 1.1.3. Suppose that the two dice of Example 1.1.2 are rolled $N = 400$ times. Let B be the event that the sum of the dice is 7. Suppose that event B occurs $f = 60$ times. The relative frequency of B is $f/N = 60/400 = 0.15$. We would expect the probability p of B to be close to 0.15.

Note. "The primary purpose of having a mathematical theory of statistics is to provide mathematical models for random experiments." [page 3 of the book] Once the probabilities are assigned to the events, the mathematical theory of probability will be well defined, even though there may be an ongoing discussion about the assigned probabilities. The mathematical theory is resented in the language of sets and functions defined on sets, so we now explore these topics.

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