Section 4.4. Order Statistics

Note. An order statistics is based on ordering a sample from lesser to greater values. In this section, we define quantiles and consider confidence intervals for quantiles.

Definition. Let $X_1, X_2, \ldots X_n$ be a random sample from a distribution of a continuous random variable X with probability density function $f(x)$ that has support $S = (a, b)$, where $-\infty \le a < b \le \infty$. Let Y_1 be the least of the S_i, let Y_2 be the next least X_i, $\ldots$ and let Y_n be the greatest X_i. Then $Y_1 < Y_2 < \cdots < Y_n$ (since X is a continuous random variable then the probability of $X_i = X_j$ for $i \ne j$ is 0). We call Y_i for $i = 1, 2, \ldots, n$, the ith *order statistic* of random sample $X_1, X_2, \ldots, X_n$.

Theorem 4.4.1. Let $Y_1, Y_2, \ldots, Y_n$ denote the n order statistics based on a random sample $X_1, X_2, \ldots, X_n$ from a continuous distribution with probability density function $f(x)$ and support (a, b). The joint probability density function of $Y_1, Y_2, \ldots, Y_n$ is given by

$$g(y_1, y_2, \ldots, y_n) = \begin{cases} n! f(y_1) f(y_2) \cdots f(y_n) & \text{for } a < y_1 < y_2 < \cdots < y_n b \\ 0 & \text{elsewhere.} \end{cases}$$