Section 4.5. Introduction to Hypothesis Testing

Note. We now turn our attention from confidence intervals to hypothesis testing. Recall that, as shown in Sections 4.1 to 4.3, a $(1-\alpha) \times 100\%$ confidence interval for a parameter θ of a population contains that parameter with a probability of $(1-\alpha)$ and does not contain the parameter θ with probability α. In this section we use these properties to state a hypothesis about the value of the parameter θ and then put a probability on that hypothesis. This is the nature of a hypothesis test, the associated probability puts a level of confidence on the hypothesis, and since these hypotheses have probabilities associated with them, then we can associate a probability with the possible errors for the hypotheses.

Note. In practice, we take a sample $X_1, X_2, \ldots, X_n$ from the distribution and compute a probability that this parameter θ is in set ω_0. This is a "hypothesis of equality" and, in practice, we desire to reject the hypothesis H_0 and, therefore, accept the alternative hypothesis H_1. Since this decision is based on probabilities we can make errors.

Definition. Let X be a random variable with density function $f(x;\theta)$ where $\theta \in \Omega$. Let ω_0 and ω_1 be disjoint subsets of Ω where $\omega_0 \,\dot\cup\, \omega_1 = \Omega$. We introduce

$$H_0 : \theta \in \omega_0 \text{ and } H_1 : \theta \in \omega_1.$$

The hypothesis H_0 is the *null hypothesis* and H_1 is the *alternative hypothesis*.

Definition. If, based on a sample $X_1, X_2, \ldots, X_n$ from the distribution of X, we decide that parameter $\theta \in \omega_1$ (where θ takes on values in Ω and $\Omega = \omega_0 \mathbin{\dot\cup} \omega_1$) when in fact $\theta \in \omega_0$, we have made a *Type I error*. If we decide $\theta \in \omega_0$ when in fact $\theta \in \omega_1$, we have made a *Type II error*.

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