

*Introduction to Mathematical Statistics* 8th Edition, Hogg,  
McKean, Craig (Pearson, 2019)  
Study Guide for Chapter 1, Probability and Distributions

The following is a *brief* list of topics covered in Chapter 1 of Hogg, McKean, and Craig's *Introduction to Mathematical Statistics*, 8th edition. This list is not meant to be comprehensive, but only gives a list of several important topics.

**1.1. Introduction.**

Random experiment, sample space, event, frequency, relative frequency.

**1.2. Sets.**

Countable set, complement of an event, subset, equal sets, union, intersection, disjoint sets, disjoint union, distributive laws (Theorem 1.2.A), De Morgan's Laws (Theorem 1.2.B), union and intersection of arbitrary collections of sets, monotone sequences of sets, limits of monotone sequences of sets, set function.

**1.3. The Probability Set Function.**

$\sigma$ -field (or  $\sigma$ -algebra) of sets, Lebesgue measure, probability set function, countable additivity, mutually exclusive collection of events, exhaustive/partition, finite additivity, monotonicity (Theorem 1.3.3), equilikely, multiplicative rule ("Rule 1"), permutation ("Rule 2"), The Birthday Problem, combination ("Rule 3"), continuity of the probability function (Theorem 1.3.6), countable subadditivity (Boole's Inequality, Theorem 1.3.7), Inclusion Exclusion Formula (Theorem 1.3.A and Theorem 1.3.B), Bonderroni's Inequality.

**1.4. Conditional Probability and Independence.**

Conditional probability, Borel sets, multiplication rule, Law of Total Probability (Theorem 1.4.B), Baye's Theorem (Theorem 1.4.1), prior probabilities, posterior probabilities, false positive, independent events, mutually independent events, The Money Hall Problem.

**1.5. Random Variables.**

Random variable, space/range, discrete/continuous random variable, probability mass function, probability density function (pdf), cumulative distribution function (cdf), random variables "equal in distribution," properties of a cumulative distribution function (Theorem 1.5.1), probabilities based on the cumulative distribution function (Theorem 1.5.2 and Theorem 1.5.3)

## **1.6. Discrete Random Variables.**

Discrete random variable, example of a possible event of probability 0, probability mass function, support of a discrete random variable, transformation of a random variable.

## **1.7. Continuous Random Variables.**

Continuous random variable, probability density function (pdf), support of a continuous random variable, absolutely continuous function/indefinite integral/“Theorem 6.11,” quantile of order  $p$ , median, first and third quantile, interquartile range, Cumulative Distribution Function Technique (Theorem 1.7.1), Jacobian, a cumulative distribution function which is a mixture of continuous and discrete types (Example 1.7.7).

## **1.8. Expectation of a Random Variable.**

Expectation (expected value or mean), of a random variable, fair game, linearity of the expectation operator (Theorem 1.8.2).

## **1.9. Some Special Expectations.**

Mean value of a random variable, variance and standard deviation of a random variable, moment generating function (mgf) of a random variable,  $m$ th moment, the use of series to find moments (Example 1.9.7), Fourier transform and inverse Fourier transform, characteristic function of the distribution of  $X$ , Laplace transform.

## **1.10. Important Inequalities.**

Markov's Inequality (Theorem 1.10.2), Chebyshev's Inequality (Theorem 1.10.3), sharpness of Chebyshev's Inequality (Example 1.10.2), convex/strictly convex function, relationships of the first derivative and second derivative of a convex function (Theorem 1.10.4), Jensen's Inequality (Theorem 1.10.5), arithmetic mean, geometric mean, harmonic mean.