Introduction to Mathematical Statistics 8th Edition, Hogg, McKean, Craig (Pearson, 2019)

Study Guide for Chapter 3, Some Special Distributions

The following is a *brief* list of topics covered in Chapter 3 of Hogg, Mckean, and Craig's *Introduction to Mathematical Statistics*, 8th edition. This list is not meant to be comprehensive, but only gives a list of several important topics.

3.1. The Binomial and Related Distributions.

Bernoulli experiment, Bernoulli trials, Jacob Bernoulli, Bernoulli distribution, "failure," "success," probability mass function for Bernoulli distribution, binomial distribution and parameters, binomial probability mass function, the moment generating function for a binomial distribution, mean and variance of a binomial distribution, relative frequency, "almost surely," the sum of independent binomial random variables has a binomial distribution, negative binomial distribution, geometric distribution, multinomial distribution, trinomial distribution, conditional distribution, hypergeometric distribution, mean and variance of hypergeometric distribution.

3.2. The Poisson Distribution.

Poisson distribution with parameter λ, Siméon Poisson, the axioms of a Poisson distribution and my additions (Note 3.2.A), derivation of the Poisson distribution from the axioms based on induction and solving differential equations (Notes 3.2.B and 3.2.C), moment generating function of a Poisson distribution, the sum of independent Poisson random variables has a Poisson distribution, applications of the Poisson distribution (Exercise 3.2.13).

3.3. The Γ, χ^2, and β Distributions.

Gamma function, the integral $\int_0^\infty y^{n-1}e^{-y}\,dy$ converges (Notes 3.3.A and 3.3.B), $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$ for $\alpha > 0$ (by Integration by Parts), definition of $\Gamma(z)$ for z complex and the Euler constant, Γ-distribution, with $\alpha > 0$ and $\beta > 0$, the moment generating function of the Γ distribution (Note 3.3.C), the mean and variance of the Γ-distribution (Note 3.3.C), hazard function, exponential distribution, $\Gamma(\alpha, \beta)$ is summable in α (Theorem 3.3.1), Γ distributions in Poisson processes (Note 3.3.D), waiting times, χ^2 distribution and probability function and degrees of freedom, the moment generating function of a χ^2 distribution and mean and variance, moment generating function of $\chi^2(r)$ (Theorem 3.3.2), a sum of independent χ^2 random variables is a χ^2 distribution (Corollary 3.3.1), the joint probability density function of two Γ distributions and the use of transformations and Jacobians to find a marginal distribution (Note 3.3.E), β distributions, mean and variance of a β distribution, Dirichlet distribution (Example 3.3.6), Dirichlet probability density function,

3.4. The Normal Distribution.

The use of polar coordinates to show the area under the standard normal distribution is 1, the moment generating function for the standard normal distribution (Note 3.4.A), mean and variance of a standard normal distribution, the normal distribution $N(\mu, \sigma^2)$, the moment generating function for $N(\mu, \sigma^2)$ (Note 3.4.B), an alternating series representation for areas under the standard normal distribution, the cumulative distribution Φ for the standard normal distribution and the z-table, the moments of $N(\mu, \sigma^2)$ (Example 3.4.4), the relationship between the square of a normal random variable and a χ^2 distribution (Theorem 3.4.1), the sum of independent normal random variables is a normal distribution (Theorem 3.4.2), the average of iid random variables has a normal distribution (Corollary 3.4.1).

3.5. The Multivariate Normal Distribution.

Bivariate normal distribution, if (X, Y) has a bivariate normal distribution then X and Y are are independent if and only they are uncorrelated (Lemma 3.5.A), the moment generating function for a multivariate normal distribution (Note 3.5.C), diagonalizability of real symmetric matrices ("spectral decomposition"), orthogonal matrix, positive semidefinite matrix and its square root (Note 3.5.D), the moment generating function of a general multivariate normal distribution (Note 3.5.E), the probability density function of a general multivariate normal distribution (Note 3.5.F), relationship between a general multivariate normal distribution and a χ^2 distribution (Theorem 3.5.1), a linear transformation of a general multivariate distribution is normal (Theorem 3.5.2), partitioned matrices and normal distributions (Corollary 3.5.1), the probability density function of the bivariate normal (Example 3.5.A), principal components of a multivariate normally distributed random vector, total variation ("TV") of a random vector, the first principal component of a multivariate normal distribution, the nth principal component of a multivariate normal distribution, Principal Component Analysis ("PCA").

3.6. t- and F-Distributions.

The joint probability density function of a standard normal distribution and a χ^2 distribution and the use of the transformation technique to derive the t-distribution as a marginal probability function (Note 3.6.A), William S. Gosset and the Student's t story, the moments and mean and variance of a t-distribution (Example 3.6.1), the use of the transformation technique to derive the F-distribution as a marginal probability function (Note 3.6.B), the mean of the F-distribution, Student's Theorem (Theorem 3.6.1).

3.7. Mixture Distributions.

Mixture distributions and mixing probabilities, the mean and variance of a mixture distribution, loggamma probability density function $\log \Gamma(\alpha, \beta)$, a mixture of uncountable infinity of distributions (see Examples 3.7.2 and 3.7.4), Pareto distribution and generalized Pareto distribution (Example 3.7.4), the transformed Pareto distribution (or Burr distribution).