Introduction to Mathematical Statistics 8th Edition, Hogg, McKean, Craig (Pearson, 2019)

Study Guide for Chapter 5, Consistency and Limiting Distributions

The following is a *brief* list of topics covered in Chapter 5 of Hogg, Mckean, and Craig's *Introduction to Mathematical Statistics*, 8th edition. This list is not meant to be comprehensive, but only gives a list of several important topics.

5.1. Convergence in Probability.

Convergence of a sequence of random variables in probability, almost everywhere/almost surely convergence, Weak Law of Large Numbers, sample mean, other versions of the Law of Large Numbers (Kolmogorov Strong Law of Large Numbers, and the Strong Lase of Large Numbers iid Case), the "linearity" of of convergence in probability (Theorems 5.1.2 and 5.1.3), convergence in probability and continuous functions (Theorem 5.1.4), convergence in probability and products (Theorem 5.1.5), random sample, unbiased estimators, consistent estimators, the sample variance is a consistent estimator of population variance (Theorem 5.1.B).

5.2. Convergence in Distribution.

Convergence of a sequence of random variables in distribution (asymptotic distribution/limiting distribution), convergence in distribution does not imply convergence in probability (Example 5.2.B), use of the Lebesgue Dominated Convergence Theorem in showing convergence in distribution (Example 5.2.3), $\overline{\lim}$ and $\underline{\lim}$ of a sequence of real numbers, convergence in probability implies convergence in distribution (Theorem 5.2.1), special cases when convergence in distribution implies convergence in probability (Theorem 5.2.2, Theorem 5.2.3, and Lemma 5.2.A), convergence in distribution and continuous functions (Theorem 5.2.4), Slutsky's Theorem (Theorem 5.2.5), bounded in probability sequence of random variables, convergence in distribution implies bounded in probability (Theorem 5.2.6), the product of a sequence bounded in probability and a sequence that converges to 0 (Theorem 5.2.7), versions of the Mean Value Theorem (Theorem 2.5.1 of Lehmann, Theorem 5.2.A A General Mean Value Theorem, and Taylor or the General Mean Value Theorem), little-o_p, big-O_p, the Δ-Method (Theorem 5.2.9), moment generating functions and convergence in distribution (Theorem 5.2.10) and its application (Example 5.2.7).

5.3. Central Limit Theorem.

Central Limit Theorem (Theorem 5.3.1) and the partial proof which assumes the existence of a moment generating function for the distribution of the population from which samples are taken, A k-Dimensional Central Limit Theorem (Theorem 5.3.A), Large Sample inference for μ (Example 5.3.1), Normal Approximation to the Binomial Distribution (Examples 5.3.3 and 5.3.4), Large Sample Inference for Proportions (Example 5.3.5), Large Sample Inference for χ^2-Tests (Example 5.3.6).

5.4. Extensions to Multivariate Distributions.

Norm on $\mathbb{R}^p$, standard basis for $\mathbb{R}^p$, inequalities involving absolute values of components and norms of vectors (Lemma 5.4.1), convergence in probability of a sequence of random vectors, convergence in probability of vectors is equivalent to componentwise convergence in probability (Theorem 5.4.1), consistent estimator, consistent estimators of variance and covariance, convergence in distribution, the interaction of convergence in distribution and continuous functions (Theorem 5.4.2), equivalence of convergence in distribution and convergence of moment generating functions (Theorem 5.4.3), Multivariate Central Limit Theorem (Theorem 5.4.4), the interaction of convergence in distribution to a multivariate normal distribution and linear transformations (Theorem 5.4.5), the interaction of convergence in distribution to a multivariate normal distribution with a continuous transformation (with continuous first partials; Theorem 5.4.6).