

Introduction to Algebra, MATH 5127

Homework 1, Section 0

Due Friday January 25, 2013 at 2:30

- 0.14a.** Show that $[0, 1]$ and $[0, 2]$ have the same cardinality by giving a formula for a one-to-one function f mapping $[0, 1]$ onto $[0, 2]$. Confirm that your function is one-to-one and onto using the definitions of “one-to-one” and “onto.”
- 0.17.** Let A be a finite set with $|A| = s$. Consider Exercise 0.16 and make a conjecture about the value of the cardinality of the power set $\mathcal{P}(A)$. Prove your conjecture using Mathematical Induction.
- 0.34.** Determine whether “ $n \mathcal{R} m$ in \mathbb{Z}^+ if n and m have the same final digit in the usual base ten notation” is an equivalence relation. That is, check if \mathcal{R} is reflexive, symmetric, and transitive. If so, describe the partition arising from the equivalence relation.
- 0.18.** For any set A , finite or infinite, let B^A be the set of all functions mapping A into the set $B = \{0, 1\}$. Show that the cardinality of B^A is the same as the cardinality of the power set $\mathcal{P}(A)$. HINT: Each element of B^A (which assigns to each element of A , either the value 0 or 1) determines a subset A in a “natural” (hint, hint: 1 in, 0 out).