Introduction to Algebra, MATH 5127

Homework 2, Section I.1 (revised)

Due Friday February 1, 2013 at 2:30

I.1.19. Find all solutions in $\mathbb{C}$ of $z^3 = -27i$. Use the polar form of a complex number, similar to the computation of roots of unity on page 18. Evaluate your solutions in terms of real and imaginary parts and evaluate all trigonometric functions.

I.1.33. Find all solutions x of the equation $x +_{12} x = 2$ in $\mathbb{Z}_{12}$ (that is, the equation $x + x \equiv 2 \pmod{12}$). Insure that you have found all solutions by checking all elements of $\mathbb{Z}_{12}$ (and showing your work).

I.1.36. There is an isomorphism of U_7 with $\mathbb{Z}_7$ in which $\zeta = e^{i(2\pi/7)} \leftrightarrow 4$. Find the element in $\mathbb{Z}_7$ to which ζ^m must correspond for $m = 0$, 2, 3, 4, 5, and 6. Examples of isomorphisms are given on pages 16, 17, and 18. We'll formally define an isomorphism as a mapping from U_7 to $\mathbb{Z}_7$ (one to one and onto, in fact) where the multiplication of two elements in U_7 behaves the same way as the addition of the corresponding elements in $\mathbb{Z}_7$. For example

$$\zeta \cdot \zeta = e^{i(2\pi/7)} \cdot e^{i(2\pi/7)} = e^{i(4\pi/7)} = \zeta^2$$

in U_7, and $4 +_7 4 = 1$ in $\mathbb{Z}_7$, so $\zeta^2 = e^{i(4\pi/7)}$ corresponds to 1: $\zeta^2 \leftrightarrow 1$. HINT: Consider the multiples of 4 in $\mathbb{Z}_7$ and the corresponding powers of $\zeta = e^{i(2\pi/7)}$ in U_7.

I.1.37. Why can there be no isomorphism of U_6 with $\mathbb{Z}_6$ in which $\zeta = e^{i(\pi/3)}$ corresponds to 4? HINT: $4 +_6 4 +_6 4 +_6 4 = 4$ in $\mathbb{Z}_6$.