Introduction to Algebra, MATH 5127

Homework 5, Sections I.6, I.7, and II.8

Due Friday March 1, 2013 at 2:30

6.37. Give an example of a finite cyclic group G having four generators, or explain why such a group does not exist. Write in complete sentences, explain your argument, and quote results from the text to justify your answer. Don't believe everything you read in the "Answers to Odd-Numbered Exercises...").

6.52. Let p be a prime number. Find the number of generators of the cyclic group $\mathbb{Z}_{p^r}$, where $r \in \mathbb{N}$. HINT: Find the elements of $\mathbb{Z}_{p^r}$ which are *not* generators.

7.3. List the elements of the subgroup generated by the subset $\{8, 10\}$ of $\mathbb{Z}_{18}$. Give a way to generate all elements of the subgroup and explain why the elements of $\mathbb{Z}_{18}$ which are not in the subgroup (if any) are not in the subgroup.

7.9. Give the table for the group having the Cayley Digraph given in Figure 7.13(b). Take e as the identity element. List the identity first in your table, and list the remaining elements alphabetically. Explain how you compute the entries of each column of the table.

8.4. Consider the permutations in S_6:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix} \text{ and } \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}.$$

Calculate $\sigma^{-2}\tau$.

8.18a. Consider the group S_3 of Example 8.7 on page 79. Find the cyclic subgroups $\langle \rho_1 \rangle$, $\langle \rho_2 \rangle$, and $\langle \mu_1 \rangle$ of S_3.

8.52. Let G be a group. Consider the functions $\rho_a : G \to G$ where $\rho_a(x) = xa$ for $a \in G$ and $x \in G$.

(a) Prove that each ρ_a is a permutation of set G (that is, it a is one to one and onto function from G to G).

(b) Prove that $P = \{\rho_a \mid a \in G\}$ forms a group under function composition (that is, show closure, associativity, identity, and inverses).

(c) Prove that the group of part (b) is isomorphic to group G.