Introduction to Algebra, MATH 5127

Homework 6, Section II.9

Due Friday March 8, 2013 at 2:30

9.3. Find all orbits of the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 5 & 1 & 4 & 6 & 8 & 7 \end{pmatrix}.$$

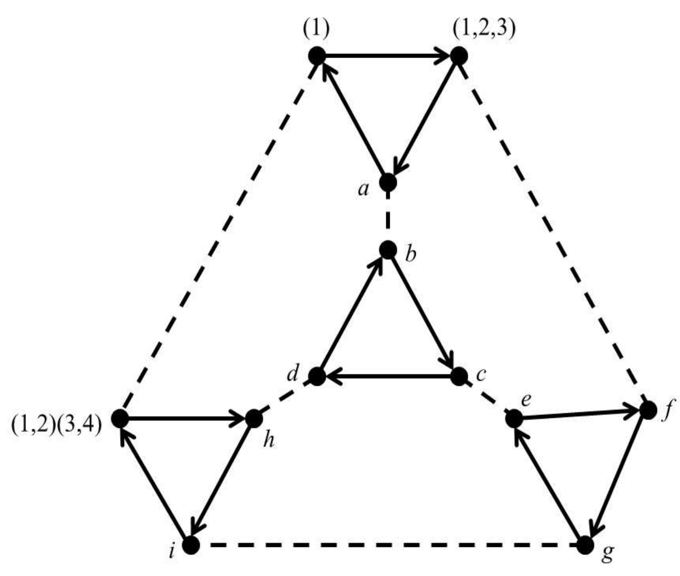
Show your work.

9.12. Express

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}$$

as a product of disjoint cycles and then as a product of transpositions.

9.19. The figure below shows a Cayley digraph for the alternating group A_4 using the generating set $S = \{(1, 2, 3), (1, 2)(3, 4)\}$. This particular graph is a "truncated tetrahedron." Label the other nine vertices with the elements of A_4, expressed as a product of disjoint cycles. Show all work in terms of how the vertex labels are computed and compute them in alphabetic order based on the labels given.



9.29. Prove that for every subgroup H of S_n $(n \geq 2)$, either all the permutations in H are even or exactly half of them are even. HINT: Let σ be an odd permutation in H (if such exists), and define $\phi : H \to H$ as $\phi(h) = \sigma h$ for $h \in H$. Show that ϕ is one to one and onto H. Count the even and odd permutations in $\phi[H]$.

9.39. Prove that S_n is generated by $\{(1,2),(1,2,\ldots,n)\}$. HINT: Show that as r varies, $\sigma = (1,2,\ldots,n)^r(1,2)(1,2,\ldots,n)^{n-r}$ gives all transpositions in S_n of the form $(1,2)$, $(2,3)$, $\ldots$, $(n-1,n)$, $(n,1)$. Then show that any transposition is a product of transpositions of this special form. NOTE: Since this gives a generating set of S_n of size two, we could use this to produce a Cayley digraph of S_n.

9.27(a). **(Bonus)** Every permutation in S_n $(n \geq 3)$ can be written as a product of at most $n-1$ transpositions. HINT: On page 90, a cycle is written as a product of transpositions.