

# Introduction to Algebra, MATH 4127

## Homework 6, Section II.9

Due Friday March 8, 2013 at 2:30

9.3. Find all orbits of the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 5 & 1 & 4 & 6 & 8 & 7 \end{pmatrix}.$$

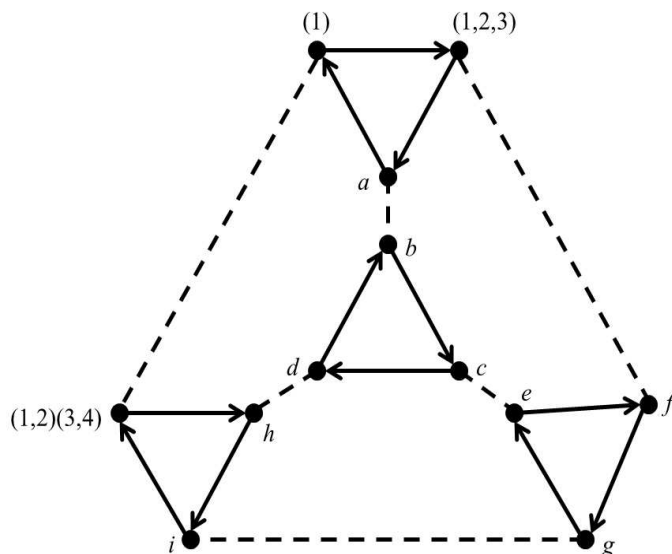
Show your work.

9.12. Express

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 4 & 7 & 2 & 5 & 8 & 6 \end{pmatrix}$$

as a product of disjoint cycles and then as a product of transpositions.

9.19. The figure below shows a Cayley digraph for the alternating group  $A_4$  using the generating set  $S = \{(1, 2, 3), (1, 2)(3, 4)\}$ . This particular graph is a “truncated tetrahedron.” Label the other nine vertices with the elements of  $A_4$ , expressed as a product of disjoint cycles. Show all work in terms of how the vertex labels are computed and compute them in alphabetic order based on the labels given.



- 9.29.** Prove that for every subgroup  $H$  of  $S_n$  ( $n \geq 2$ ), either all the permutations in  $H$  are even or exactly half of them are even. HINT: Let  $\sigma$  be an odd permutation in  $H$  (if such exists), and define  $\phi : H \rightarrow H$  as  $\phi(h) = \sigma h$  for  $h \in H$ . Show that  $\phi$  is one to one and onto  $H$ . Count the even and odd permutations in  $\phi[H]$ .
- 9.27(a). (Bonus)** Every permutation in  $S_n$  ( $n \geq 3$ ) can be written as a product of at most  $n - 1$  transpositions. HINT: On page 90, a cycle is written as a product of transpositions.