Introduction to Algebra, MATH 5127

Homework 7, Section II.10

Due Friday March 22, 2013 at 2:30

10.6 & 7. Recall that $H = \{\rho_0, \mu_2\}$ is a subgroup of D_4 (see Table 8.12 on page 80). Find all left cosets and right cosets of H. Are the left cosets the same as the right cosets? Show your computations for each coset.

10.13. (Modified from the text's version.) Consider the subgroup $H = \langle \rho_1 \rangle$ of S_3. Find the index of the subgroup in S_3, $(S_3 : H)$. Use Table 8.8 on page 79 and explain your reasoning.

10.35. Let H be a subgroup of G. Prove that the cardinality of the set of left cosets of H is the same as the cardinality of the right cosets of H. That is, find a one to one and onto mapping ϕ from the set of left cosets of H, $H_{\mathcal{L}}$, to the set of right cosets of H, $H_{\mathcal{R}}$. *Prove* that ϕ is one to one and onto. HINT: Define $\phi(aH) = Ha^{-1}$. Confirm that ϕ is well defined. Notice that $H = \{h^{-1} \mid h \in H\}$.

10.45. Prove that a finite cyclic group of order n has exactly one subgroup of each order d dividing n. Prove that there are no other subgroups. HINT: Lagrange's Theorem.