Review Notes. For Sections X.50 and X.51

Note. The following results will be useful to recall when covering Sections X.50 and X.51.

Theorem 30.23. Let E be an extension field of field F and let $\alpha \in E$ be algebraic over F. If $\deg(\alpha, F) = n$, then $F(\alpha)$ is an n-dimensional vector space over F with basis $\{1, \alpha, \alpha^2, \ldots, \alpha^{n-1}\}$. Furthermore, every element β of $F(\alpha)$ is algebraic over F, and $\deg(\beta, F) \leq \deg(\alpha, F)$.

Definition 31.2. If an extension field E of field F is of finite dimension n as a vector space over F, then E is a *finite extension of degree n over F.* We denote this as $n = [E : F]$.

Theorem 31.3. A finite (degree) extension field E of field F is an algebraic extension of F.

Definition 48.16. The group $G(E/F)$ of Theorem 48.15 is the group of *automorphisms of E leaving F fixed*, or the *group of E over F*.

Definition 49.9. Let E be a finite extension of a field F. The number of isomorphisms of E onto a subfield of $\overline{F}$ leaving F fixed (which is finite by Theorem 49.7) is the *index of E over F*, denoted $\{E : F\}$.

Corollary 50.7. If $E \leq \overline{F}$ is a splitting field over F, then every isomorphic mapping of E onto a subfield of $\overline{F}$ leaving F fixed is actually an automorphism of E. In particular, if E is a splitting field of finite degree over F, then $\{E : F\} = |G(E/F)|$, where $G(E/F)$ is the group of automorphisms of E having F fixed.

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